Himachal Pradesh University

Department of Mathematics and Statistics

M.Phil. / Ph. D Course Work (Mathematics)

First Semester: Syllabi

w.e.f. October 2018 (Session 2018-2019 Onwards)

Total Duration: One Year (Two Semesters, Second semester: Dissertation work)

The syllabus of the 1st semester is common with Ph. D Course work, applicable for those students admitted in Ph. D without having M.Phil. degree.

Course-A (Compulsory)

Research Methodology for Mathematical Sciences

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

NOTE: Eight questions, each of 14 marks, will be set in the question paper and the candidate shall be required to attempt any five questions

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Research: Definition, Concepts and General introduction, Different approaches to research, Basic, Applied, Interdisciplinary, Multidisciplinary research.

Hypothesis, Theories, Laws, Mathematics & Science.

Scientific Methods: Examples of the scientific methods. The role of computers in scientific process, and use of computers in obtaining proofs of mathematical results.

Problem Posing: The soul of research methodology. The art of solving problems, Polya's scheme for solving problems. General methods for solving problems.

Research Tools: Introduction to Type-Settings, MS-Office, Scientific work place, Latex. Introduction to MATHEMATICA, MATLAB, SCI LAB. Knowledge of online e-resources, online and open access journals, Math. Sci. Net., Zentralblatt Math, Scopus, JSTOR, Google Scholar, Research Gate, Concept of citation index, impact factor, H-index.

Scientific Writing: Writing a paper for conference and journals. Communicating research. Obtaining offprints of papers. Preparing M.Phil. Dissertation and Ph.D Thesis.

Ethics and responsibility in scientific research: Ethics, Western and Eastern perspective on the source of Ethics, Rights and Permissions.

Recommended Text Books:

1. Research Methodology: The aims Practices and Ethics of Science, Peter Pruzen, Springer, (2016). Chapter: 2; (2.8), Chapters: 3, 9 &10.
2. Research Methodology for scientist and Engineers, J. N. Kapur, Mathematical Science Trust Society, (1997). Chapters 2, 3, 6, 7 & 12.
3. How to write and publish a scientific paper, Robert A. Day, Cambridge University Press Fourth Edition (1996). Chapters 1,2,3, 20, 23& 26.

Optional Courses: Student have to opt for any two of the following Courses:(I to VIII)

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Course-I

Hydrodynamic and Hydromagnetic Stability-I

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

NOTE: Eight questions, each of 14 marks, will be set in the question paper and the candidate shall be required to attempt any five questions

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Basic Concepts: Introduction; Basic concepts; The analysis in terms of normal modes; Non-dimensional number.

The Thermal Instability of a Layer of Fluid Heated From Below: The Bénard Problem; The basic hydrodynamic equations; The Boussinesq approximation; The perturbation equations; The analysis into normal modes; The principle of exchange of stabilities; The equations governing the marginal state and reduction to a characteristic value problem; Exact solution of the characteristic value problem (when instability sets in as stationary convection), the case of two free boundaries.

The Thermal Instability of a Layer of Fluid Heated From Below: *The Effect of Rotation*: the problem of thermal instability in a rotating fluid, The perturbation equations; The analysis into normal modes. The case when instability sets in as stationary convection, A Variational Principle; Solutions for the case when instability sets in as stationary convection, the case of two free boundaries; On the onset of convection as over stability, the solution for the case of two free boundaries.

The Thermal Instability of a Layer of Fluid Heated From Below: ***The Effect of a Magnetic Field:*** The problem of thermal instability in the presence of magnetic field; The perturbation equations; The case when instability sets in as stationary convection, A variational principle; Solutions for when instability sets in as stationary convection, the case of two free boundaries; Onset of Convection as over stability.

The Stability of Superposed Fluids: ***The Rayleigh-Taylor instability***: Introduction; The character of the equilibrium of a stratified heterogenous fluid, The perturbation equations; The Inviscid case; The effect of rotation; The effect of vertical magnetic field.

Recommended Text Book:

S. Chandrasekhar: Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York, 1981.

Chapter 1: § 1 to 4.
Chapter II: § 5 to 12 & §15(a).
Chapter III: § 24, 25, 26, 27(a), and § 29.
Chapter IV: § 41, 42, 43(a), § 44(a) and § 46.
Chapter X: § 90, 91, 92, 95 and § 96.

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Course-II

Hydrodynamic and Hydromagnetic Stability-II

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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Initiation of Magnetoconvection

Review of the simple Bénard instability problem, Magnetohydrodynamic simple Bénard instability problem, The governing equations and Thompson's condition for the Exchange Principle. Extension of viscous case and Chandrasekhar's first method, Chandrasekhar's second method and his conjecture, A Sufficient condition for the exchange principle, Resolutions of Chandrasekhar's conjecture concerning the two energies, Solutions for the case when exchange principle is valid. Solutions for the case when over stability is valid, settlement of the recent controversy, Some illustrative examples.

Reformulation of the Simple Bénard and Thermohaline Instability Problem

Basis of the modified theory, Inadequacy of the classical theory, Construction of the modified, simplified governing equations, Modified equations for thermohaline instability problem, Modified Analysis of Simple Bénard instability problem and thermohaline instability problem, The eigenvalue

problem, Characterization of the marginal state and the marginal state solution, Some illustrative examples.

Limitations of the Complex Wave Velocity in the Instability Problem of Heterogeneous Shear

Introduction, Governing Equations and initial stationary state solution, The perturbation equations, The normal mode analysis, The Mathematical eigenvalue problem and classification of modes, The origin of Kelvin-Helmholtz instability and Taylor's conjecture: Heuristic considerations The works of (a) Synge (1933), (b) Miles (1961), (c) Howard (1961). The problem of simultaneous reduction and unification, the work of Banerjee and Jain, A reduction theorem, the work of Kochar and Jain, An illustrative example.

Text book:

1. Hydrodynamic and Hydromagnetic Stability, Mihir B. Banerjee and Jagdish, R. Gupta, Silver Line Publishers.
   Chapter-I: § 1.1 to 1.9.
   Chapter-II: § 2.1 to 2.8.
   Chapter-III § 3.1 to 3.10.

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Course-III

Fluid Flow Instability, MHD, Plasmas and Geophysical Fluid Dynamics

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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The Stability of Superposed Fluids: The Kelvin-Helmohlotz Instability

The Perturbation equations and boundary conditions. The case of two uniform fluids in relative horizontal motion separated by a horizontal boundary. Discussions in the absence and presence of surface tension. The effect of rotation. The effect of a horizontal magnetic field.

MHD and Plasmas

Magnetohydrodynamics (MHD). Introduction. Maxwell's equations for moving media, Magnetic induction equation and Maxwell's equations. Basic equations of MHD.

Motion Of A Charged Particle

General characteristics. The equations of motion of a charged particle in crossed electric and magnetic fields. The motion of a charged particle in a uniform magnetic field.

Geophysical Fluid Dynamics

Definition of porous medium. Porosity. Methods for measurement of porosity. Flow of homogeneous fluids in porous media. Darcy's law. Darcy's Oberbeck- Boussinesq (DOB) equations for material. Darcy's law further generalized. Basic equations of flow through porous media.

Text Books

1. Hydrodynamics and Hydromagnetic Stability, S. Chandrasekhar, Dover Publications, New York (1981), Chapter XI: §§100,101,105,106
2. Stability of Fluid Motions II, D.D. Joseph, Springer-Verlag, New York (1976).
3. An Introduction to Magneto-Fluid Mechanics, V.C.A. Farraro and C. Plumpton, Oxford University Press (1966). Chapter 7: §§ 7.1-7.4
4. The Physics of Flow Through Porous Media, A.E. Schidegger, University of Toronto Press, Toronto (1974).

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Course-IV
Groups Rings and Modules

Time Allotted:3 Hours. ***Maximum Marks: 70***
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***GROUPS* & Ideals:**

Characters of finite abelian groups, The Character group, the orthogonality relations for characters, Maximal Ideal, Generators, Basic Properties of Ideals, Algebra of Ideals, Quotient Rings, Ideal in Quotient Rings, Local Rings.

The Jacobson Radical

Modules; The radical of a ring, Artinian rings, Semisimple Artinian rings.

Semi simple Rings

The density theorem, Semi simple rings, Applications of Wedderburn's theorem.

Text Books:

1. Introduction to Analytic Number Theory, Tom M. Apostol., Narosa Publishing House, New Delhi, Chapter-VI: Pages 129 to 136.
2. Non-Commutative Rings, I.N. Herstein, John Wiley and Sons, Inc., Chapters: I&II, pages 1 to 68.
3. Introduction to Rings and Modules 2nd Edition, C. Musili, Narosa Publishing House, New Delhi; Chapter-II: Pages 33 to 65.

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Course-V

Advanced Matrix Analysis

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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Properties and characterizations of Hermitian matrices, Variational characterization of eigenvalues of Hermitian matrices. Rayleigh-Ritz theorem, Courant-Fischer theorem (Min.-Max. Principle), Some applications of the variational characterization, Weyl theorem, Schur majorization theorem, Interlacing theorem, Inclusion principle.

The field of values or Numerical range, Subadditivity and eigenvalues of sums, stability analysis, Definitions, Basic properties of the field of values, convexity, Toeplitz-Hausdorff theorem.

The Tensor Product of matrices, Definition and Basic properties, Eigenvalues of the Tensor product of two square matrices.

The Hadamard product, some Basic observations, The Schur product theorem.

Operator Monotone and Operator Convex Functions.

Positive linear maps, Examples and properties of positive maps, Kadison's inequality, Choi's theorem, Choi's inequality, Inequality complementary to Choi's inequality, Definition and examples of completely positive linear maps.

Recommended Text Books:

1. **Matrix Analysis**: Roger A. Horn and Charles R. Johnson. Second Edition (2013), Cambridge University Press. **Chapter 4, 4.1:** Definitions 4.1.1, 4.1.9, 4.1.11, 4.1.12. Theorem 4.1.2-4.1.10. Proposition 4.1.13., **4.2:** Theorem 4.2.2, 4.2.6, 4.2.10. Lemma 4.2.3, 4.2.4 Observation 4.2.5 Corollary 4.2.12.,**4.3** Theorem 4.3.1, 4.3.17, 4.3.28, 4.3.45. Corollary 4.3.3, 4.3.5, 4.3.7, 4.3.12, 4.3.15, 4.3.34. Definition 4.3.41, 4.3.43.
2. Topics in Matrix Analysis: Roger A. Horn and Charles R. Johnson, (1991), Cambridge Univ. Press. **Chapter 1, 1.0,** 1.0.1, 1.0.2, 1.0.2.1, 1.0.3, 1.0.3.1, **1.1,** 1.1.1-1.1.4, **1.2,** 1.2.1-1.2.12, **1.3,** 1.3.3. **Chapter 4, 4.2,** 4.2.1-4.2.13, **Chapter 5, 5.0 (**5.0.1, 5.0.2, 5.0.3a, 5.0.5), **5.1 & 5.2.**
3. Matrix Analysis: Rajendra Bhatia, (1997), Springer, **Chapter V, V.I .**
4. Positive Definite Matrices: Rajendra Bhatia, (2007), Hindustan Book Agency. **Chapter 2, 2.2, 2.3** (2.3.1-2.3.6) & **2.7** (2.7.6-2.7.9)

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Course-VI

Boundary Layer Theory

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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CHAPTER-XIV: BOUNDARY-LAYER CONTROL IN LAMINAR FLOW

(A) Methods of boundary-Layer Control.

1. Motion of the solid wall. 2. Acceleration of the boundary layer (blowing).3. Suction
4. Injection of a different gas. 5. Prevention of transition by the provision of suitable shapes. Laminar aerofoils. 6. Cooling of the wall.

(B) Boundary-Layer suction.

1. Theoretical results: 1.1. Fundamental equations, 1.2 Exact solutions (Flat plate: Boundary Layer with pressure gradient) Approximate Solutions

(C) Injection of a different gas (Binary boundary Layers): 1.1 The fundamental equations

CHAPTER –XV: NON-STEADY BOUNDARY LAYERS.

(A) Basic Equations:

1. Boundary –Layer equations. 2. The Method of successive approximations.
3. C.C.Lin's Method for periodic external flows. 4. Expansion into a series when a steady stream is perturbed slightly. 5. Similar and semi-similar solutions. 6. Approximate solutions

(B) ***Boundary-Layer formation after impulsive start of motion.***

1. Two-dimensional case. 2. Axially symmetrical problem.

(C) Boundary-Layer formation in accelerated motion.

(E) Periodic boundary-layer flows.

1. Oscillating cylinder in fluid at rest. 2. C.C. Lin's theory of harmonic oscillations.
3. External flow with small, harmonic perturbation boundary-layer theory.

PERTURBATION METHODS

Parameter perturbation, co-ordinate perturbation, order symbols and Gauge functions, asymptotic expansions and sequences. Convergent versus Asymptotic series, Non Uniform expansions.

Books:

1. Boundary-Layer Theory, Dr. Hermann Schlichting, Mc GRAW-HILL Book Company, New York.
2. Perturbation Methods, A.N. Nayfeh, A Willey-Inter science Publication.

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Course-VII

Cryptography

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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Mathematics of Cryptography: Integer Arithmetic, Modular Arithmetic, Matrices, Linear Congruence.

Traditional Symmetric-Key Ciphers: Substitution Ciphers, Transposition Ciphers, Stream and Block Ciphers.

Mathematics of Symmetric-Key Cryptography: Algebraic Structures, GF(2^n) Fields.

Introduction to Modern Symmetric-Key Ciphers: Modern Block Ciphers,
Modern Stream Ciphers.

Data Encryption Standard (DES): DES Structure, DES Analysis, Security of DES, Multiple DES-Conventional Encryption Algorithm.

Advanced Encryption Standard(AES) : Transformations, Key Expansion,
The AES Ciphers, Analysis of AES.

Encipherment Using Modern Symmetric-Key Ciphers: Use of Modern Block Ciphers, Use of Stream Ciphers.

Mathematics of Asymmetric-Key Cryptography: Primes, Primality Testing, Factorization, Chinese Remainder Theorem, Quadratic Congruence, Exponentiation and Logarithm.

Asymmetric-Key Cryptography: RSA Cryptosystem, Rabin Cryptosystem, ElGamal Cryptosystem, Elliptic Curve Cryptosystem.

Text Book:
Forouzan, B.A. & Mukhopadhyay, D., *" Cryptography and Network Security", Tata McGraw Hill Publication.* [Chapter 2 – Chapter 10]

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Course-VIII
Algebraic Number Theory

Time Allotted:3 Hours. ***Maximum Marks: 70***
Minimum pass marks: 35

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Algebraic numbers and algebraic integers. Countability of set of Algebraic numbers. Liouville's Theorem and generalizations . Transcendental Numbers. . Thue Theorem and Roth's theorem (statement only). Algebraic number field . Theorem of Primitive elements. Ring of algebraic integers. Norm and trace of algebraic numbers. Non degeneracy of bilinear pairing. Existence of integral basis. Discriminant of an algebraic number field. Ideals in the ring of algebraic integers. Explicit construction of integral basis. Sign of discriminant. Cyclotomic fields. Calculation for quadratic and cubic cases.

Integrally closure. Noetherian rings. Characterizing of Dedekind domains. fractional ideals and unique factorization. G.C.D. and L.C.M. of ideals. Chinese Remainder theorem. Dedekind theorem. Ramified and unramified extensions. Different of an algebraic number field. Factorization in the ring of algebraic integers.

Recommended Text Books:

1. Jody Esmonde and M.Ram Murty "Problems in Algebraic Number Theory", (Springer Verlag, 1998)
2. S Lang , Algebraic Number Theory GTM Vol- 110. Springer Verlag (1994).

Reference Books:

1. Paulo Ribenboim, Algebraic Numbers, Wiley Interscience Newyork(1972).
2. R. Narasimhan and S. Raghavan, Algebraic Number Theory, Mathematical Pamphlets-4. Tata Institute Fundamental Research (1966).