B.Ed. 1st YEAR

Paper IX – A (iii)

TEACHING OF MATHEMATICS

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TEACHING OF MATHEMATICS
Paper IX – A (iii)
Contents

SYLLABUS
Paper IX – A (iii)
TEACHING OF MATHEMATICS

Course objectives: **Marks: 50 (40 + 10)**

The student teachers will be able to:

- understand the nature and characteristics of Mathematics.
- know the importance and values of teaching mathematics.
- understand the relationship of mathematics with other subjects of the school curriculum.
- understand aims and objectives of teaching mathematics at school stage.
- state objectives in behavioural terms with reference to concepts and generalizations.
- understand the contributions made by Indian and Western mathematicians.
- apply various methods of teaching of mathematics.
- differentiate between methods and techniques of teaching mathematics.

INSTRUCTIONS FOR CANDIDATES

The question paper will consist of three sections: A, B and C. Section A will consist of 4 short answer type questions (2 marks each) which will cover the entire syllabus uniformly and carry 8 marks. Sections B and C will have two long answer type questions from the respective units 1 and 2 of the syllabus and will carry 16 marks each. Candidates are required to attempt one question each from the sections B and C of the question paper and entire Section A. Answer to short question should be completed in around 100 words each.

Unit 1: Foundations of Mathematics Education

- Mathematics: Meaning, Nature, Importance and Place of Mathematics in Daily Life and School Curriculum.
- Aims and Objectives of Teaching Mathematics: Concept and Meaning, General Aims of Teaching Mathematics, Formulation and Classification of Objectives in Behavioural terms with Reference to Cognitive, Psycho-motor and Affective Domains.
- Concept of Vedic Mathematics, Contributions of Indian Mathematicians: Bhaskaracharya, Aryabhatta and Ramanujam; Contributions of Western Mathematicians: John Venn and Pythagoras.
- Curriculum Construction in Mathematics, Selection and Organization, Factors affecting Change in Mathematics Curriculum, Process of Evaluation of Mathematics Curriculum at School Level.

Unit 2: Teaching Methods/Approaches and Techniques

- Methods of Teaching Mathematics: Project Method, Problem Solving and Laboratory Method.
- Approaches of Teaching Mathematics: Inductive-deductive, Analytic-synthetic, Heuristic.
- Techniques and strategies of teaching Mathematics: Drill and Practice, Assignment, Homework, Supervised Study, Play Way Technique, and Activity Based Technique.
- Strategies for Teaching of Mathematics to CWSN (Gifted, Slow Learners, Learners with Dyscalculia). Difficulties Faced by the Teachers in Teaching of Mathematics and Suggestive measures to overcome them.

Activity (Any One of the Following)

1. Prepare any one self-made working (3-dimensional) teaching model from locally available resources for teaching of mathematics at the senior secondary stage.

2. Power point presentation on any one topic of mathematics of standard VI to X.

3. Prepare a report on critical analysis of Mathematics curriculum prescribed by HPBSE / CBSE for secondary school stage.

Unit - 1

Meaning, Nature, Importance and Place of Mathematics in Daily Life and School Curriculum

Structure

1.1 Introduction

Dear Learner, Mathematics plays a crucial role in our daily lives and its importance cannot be overstated. It is a fundamental discipline that impacts various aspects of our everyday existence. Here are some key reasons why mathematics is important in our daily life.

- Mathematics is essential for performing everyday tasks such as counting money, measuring ingredients for cooking, telling time, and making basic calculations. Without arithmetic skills,
- it would be challenging to handle simple financial transactions.
- Mathematics equips us with problem-solving skills that are applicable in various situations.
- Whether you're trying to figure out the most cost-effective way to purchase groceries, solve a household issue, or make decisions at work, problem-solving abilities rooted in mathematics are invaluable.
- In the age of information, data analysis is increasingly important.

- ❖ Mathematics is vital for interpreting data, making sense of statistics, and drawing conclusions. This skill is valuable for understanding news, research findings, and even personal health data.
- ❖ Mathematics is the language of science and technology. It is essential for fields like engineering, physics, computer science, and medicine. Without mathematics, advancements in these fields would not be possible.
- ❖ Mathematical skills are often a prerequisite for many careers and educational opportunities. Whether you're pursuing a career in science, technology, engineering, mathematics (STEM), or even in fields like economics and business, a mathematical foundation is essential.
- ❖ Mathematics fosters critical thinking and analytical reasoning. It encourages precision and logical thought processes, which can be applied to various aspects of life, from making informed decisions to evaluating arguments and making sense of complex issues.
- ❖ Even in fields traditionally associated with creativity, such as art and design, mathematics plays a majorrole. Concepts like symmetry, geometry, and proportions are fundamental to creating aesthetically pleasing and functional designs.
- ❖ Basic mathematical concepts, such as understanding maps, distances, and angles, are crucial for navigation and travel. Whether you're planning a road trip or using GPS, mathematics helps you get from one place to another.
- ❖ Mathematics is fundamental for time management. Whether it's scheduling appointments, managing deadlines, or optimizing your daily routine, mathematical skills are invaluable for making the most of your time.

In summary, mathematics is not only a subject studied in school; it is a fundamental skill that permeates almost every aspect of our daily lives. It equips us with the tools to make informed decisions, solve problems, and navigate the complexities of our modern world. Whether you're balancing your check book, analyzing data, or pursuing a career in any subject/field, mathematics is an integral part of daily life.

1.2 Learning Objectives

After completing this unit you will be able to:

- explain the meaning of mathematics.
- discuss the nature of mathematics.
- explain the importance of mathematics.
- describe the place of mathematics in school curriculum.

1.3 Meaning of Mathematics

Dear learner, we all know what is mathematics, but when it comes to defining it we seem to fail completely. A dictionary meaning of mathematics is a science dealing with the study of quantities and their relationships expressed in numbers and other special symbols. Some people describe mathematics more as a language in which every symbol and every combination has precise meaning that can be determined by the application of logical rules. This language can be used to describe and analyse anything in the universe. It is not easy to give one definition of mathematics, but we can explain aspects of mathematics by what it does.

Definitions of Mathematics

In many Indian languages, the vernacular word for mathematics is “Ganita” which means the science of space and quantity which helps us in solving many problems of life using numeration and calculation.

In Sanskrit, it is said, “Ganita Shastra” which means the science of counting and calculation for mankind. Mathematics as Noun, functioning as singular, is group of related sciences, including algebra, geometry, and calculus, concerned with the study of numbers, quantity, shape, and space and their interrelationships by using a specialized notation. Functioning as singular or plural, mathematical operations and processes involved in the solution of a problem or study of some scientific field. A few definitions of Mathematics are given below:

“Mathematics should be visualized as the vehicle to train a child to think, reason, analyze, and articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and meaning” (National Policy on Education, 1986).

“We cannot overstress the importance of Mathematics in relation to science, education and research. This has always been so, but at no time has the significance of Mathematics been greater than what it is today**.** It is important that deliberate efforts are made to place India on the world map of Mathematics within the next two decades or so” (National Education Commission or Kothari Commission, 1964-66).

“Mathematics is a way to settle in the mind a habit of reasoning” (Locke).

“Mathematics is the study or use of numbers and shapes to calculate, represent, or describe things. Mathematics includes arithmetic, geometry, and algebra” (Chandel J.C).

“Mathematics is the mirror of civilization” (Hogben).

According to Cambridge Dictionaries Online, “Mathematics is the study of numbers, shapes, and space using reason and usually a special system of symbols and rules for organizing them”.

“It is the science of quantity” (Aristotle),

According to Scholastic Children's Dictionary (1996),**"**Mathematics is the study of numbers, quantities, shapes, and measurements and how they relate to each other."

"Mathematics is the indispensable instrument of all physical resource**s**"(Kant).

"Mathematics is a way to settle in the mind a habit of reasoning" (Locke).

"Mathematics is the gateway and key to all sciences" (Bacon).

From the above definitions, we can say that;

- Mathematics is the science of quantity and space.
- Mathematics is the science of numbers.
- Mathematics is the science of logical reasoning.
- Mathematics is the perfection of generalization
- Mathematics is the method of progress of various subjects.
- Mathematics is an important means of generalization

Mathematics is therefore is not only number work or computation, but is more about forming generalizations, seeing relationships and developing logical thinking and reasoning.

Self-Check Exercise - 1

1. Mathematics is _____

    a) Science of number and space
    b) Study of numbers and figures
    c) Science of indirect measurements
    d) All of the above

2. According to Locke "Mathematics is a way to settle in the mind a habit of _____

    a) Calculation
    b) Reasoning
    c) Simplification
    d) Analysis

1.4 Nature of Mathematics

Mathematics combines logic and creativity, serving both practical purposes and intellectual curiosity. For some, including many who are not professional mathematicians, the beauty and intellectual challenge of mathematics are at its core. For others, such as scientists and engineers, the primary value of mathematics lies in its practical application to their work. Given its crucial role in modern culture, having a basic understanding of mathematics is essential for scientific literacy. To achieve this, students must recognize mathematics as part of the scientific endeavor, grasp the nature of mathematical thinking, and familiarize themselves with

fundamental mathematical concepts and skills. The nature of mathematics can be summarized as follows:

- Mathematics is the science of quantity and space.
- Mathematics is the study of numbers.
- Mathematics is the science of logical reasoning.
- Mathematics is the art of generalization.
- Mathematics drives progress in various fields.
- Mathematics is an essential tool for generalization.
- Mathematics is inherent to human nature and empirical in nature. People naturally engage in mathematical thinking.
- Mathematics exists independently of us. We can conduct experiments, explore, and investigate, testing ideas and making decisions based on our findings.
- Mathematics is the study of structure, operations, order, shape, continuity, and transformation.
- Mathematics is the skill of problem-solving.
- Mathematics is the science of patterns and relationships. As a theoretical discipline, it explores possible relationships between abstractions, regardless of whether they relate to the real world.
- Mathematics is also an applied science. Many mathematicians focus on solving real-world problems, looking for patterns and relationships while using techniques similar to those in theoretical mathematics.
- Due to its abstract nature, mathematics is universal in ways other fields of thought are not.
- Mathematics is the primary language of science. Its symbolic language is highly valuable for clearly expressing scientific concepts.
- Mathematics provides the grammar of science, offering rules for analyzing scientific ideas and data precisely.
- Mathematics and science share several characteristics, including a belief in order, the combination of imagination and rigorous logic, the values of honesty and transparency, and the importance of peer review.
- Mathematics develops abilities in induction, deduction, and generalization.
- Mathematics is a structured, organized, and exact branch of science.

Self-Check Exercise- 2

1. What is the nature of Mathematics?

2. Mathematics develops ability of _____

    a) Induction
    b) Deduction
    c) Generalization.

d) All of the above

3. The nature of mathematics is:
   a) Ornamental
   b) Difficult
   c) Logical
   d) Unsystematic

1.5 Importance of Mathematics

“Mathematics is the mirror of civilization,” as its history reflects the development of cultures and societies. A nation’s culture and civilization are demonstrated by the mathematical knowledge it possesses. Mathematics aids in the preservation, promotion, and transmission of culture. Various forms of cultural expression, such as poetry, painting, sculpture, and music, rely on mathematical principles. Mathematics has aesthetic value, with concepts like symmetry, order, proportion, and size forming the foundation of all art and beauty. All forms of poetry and music involve mathematical elements. Therefore, teaching mathematics at all levels is crucial.

Mathematics is applied across all sciences, including biology, chemistry, and physics, as well as in social sciences like economics, psychology, and sociology. It is also essential in engineering fields such as civil, mechanical, and industrial engineering, and in technological domains like computers, rockets, and communications. Even the arts, including sculpture, drawing, and music, utilize mathematics. Anything involving a computer depends on mathematics, which is ubiquitous in today’s world. Furthermore, learning mathematics develops logical thinking and problem-solving abilities, as well as precision in communication. It imparts valuable life skills and touches nearly every aspect of life. Mathematics is all around us, and its importance continues to grow.

Mathematical skills are vital for a wide range of applications in analysis, technology, science, security, and economics. Educating students to become proficient in mathematics and appreciate its value is critical for the future. For those passionate about mathematics, sharing its significance with others is a rewarding endeavor. Everyday activities require an understanding of mathematical concepts and processes. To comprehend many materials in newspapers, a solid understanding of mathematical terminology—such as percentages, discounts, commissions, and taxes—is necessary. As society becomes increasingly complex, new terms from electronics and computing are added. Certain decisions demand a strong understanding of quantitative relationships. The ability to recognize, define, and solve problems systematically is essential. Many careers, including those in economics, architecture, engineering, aviation, and science, rely on mathematical skills.

Key aspects of mathematics include:

- Teaching how to analyze situations, make decisions, evaluate reasoning, recognize relationships, concentrate, and maintain accuracy and organization in work.
- Developing the ability to perform calculations quickly and accurately, as well as understanding measurement processes and using precise instruments.
- Cultivating skills for estimating and approximating, applying formulas, procedures, and comparison methods, representing designs and spatial relationships through drawings, organizing numerical data, and interpreting graphical or tabular information.

Applications of Mathematics:

1. **Mathematics and Society**: The contribution of mathematics to society's progress is evident in its influence on trade, commerce, banking, global marketing, construction, architecture, transportation, communication, and technological advancements. Mathematics has been the backbone of societal development from ancient times to today's digital age.
2. **Mathematics and Other School Subjects**: Mathematics plays a crucial role in teaching other subjects within the school curriculum, including languages, sciences (including social sciences), work experience, health and physical education, and the arts. The study of mathematics facilitates learning across various disciplines.
3. **Mathematics and Career Development**: The field of mathematics has expanded greatly, and mastery of mathematical skills is key to success in numerous professions. Career paths in engineering, banking, commerce, finance, accountancy, economics, business, research, teaching, statistics, astronomy, and forecasting all rely on mathematical knowledge.
4. **Careers in Mathematics**: Mathematics serves as a gateway to careers in diverse fields such as engineering, banking, commerce, finance, accountancy, economics, business, research, teaching, statistics, astrology, astronomy, and more.

Profession in Mathematics

- Teaching jobs in schools and higher academic institutions i.e. colleges and universities.
- Jobs related to Engineering
- Economist, auditor, tax consultant and corporate financial advisor.
- System analyst and operation research analyst.
- Statistician
- Research Scientist and Data Analyst
- Environmental Mathematician and Ecologist
- Geophysical Mathematician

- Chemical/physical/biological mathematician
- Computer scientist, space scientist and robotics engineer
- Astronomer and Astrologer
- Financial estimate makers and forecasting personnel
- Investment Advisors and planners.

Self-Check Exercise - 3

What is the importance of mathematics?

1.6 Place of Mathematics in Daily Life and School Curriculum

The most fascinating of all knowledge and the most phobia-generating subject in the school curriculum is Mathematics. The way in which it is introduced to the learners devoid of its daily application has been the root cause of the present scenario as experienced by everyone in the country. It is pertinent to note the ways in which one experiences mathematical applications in daily life from the time one gets up from bed till one goes to bed in the night. Mathematics is an essential and integral part of the school curriculum for several important reasons:

Critical Thinking and Problem-Solving Skills: Mathematics teaches students how to think critically, analyse problems, and develop problem-solving skills. These skills are valuable not only in mathematics but in various other aspects of life.

Quantitative Literacy: In our data-driven world, the ability to understand and work with numbers is crucial. Mathematics equips students with quantitative literacy, which is necessary for making informed decisions in various fields, including finance, science, and social sciences.

Scientific and Technological Advancements: Mathematics is the language of science and technology. Without a solid foundation in mathematics, students would struggle to comprehend and contribute to the advancements in fields such as physics, engineering, computer science, and more.

Career Opportunities: Many careers and professions require a strong background in mathematics. These include professions in science, technology, engineering, medicine, finance, and even fields like architecture and data analysis.

Logical Reasoning: Mathematics encourages logical reasoning, which is important not just for solving mathematical problems but for making rational decisions in everyday life.

Educational Foundation: Mathematics often serves as a foundation for learning other subjects. For instance, concepts in physics, chemistry, and economics are deeply rooted in mathematical principles.

Cognitive Development: Learning mathematics helps in cognitive development. It challenges the mind, encourages abstract thinking, and enhances memory and problem-solving abilities.

Real-World Applications: Mathematics is used in various real-world situations, from budgeting and personal finance to understanding probabilities and risks. A strong mathematical background is essential for making sound financial decisions.

Global Competitiveness: In a globalized world, students need to be competitive on a global scale. Proficiency in mathematics is often a benchmark for assessing a country's competitiveness in science and technology.

Personal Empowerment: Mathematics empowers individuals to understand and navigate the world around them more effectively. It can boost self-confidence and lead to a sense of achievement.

Innovation and Creativity: Mathematical thinking fosters creativity and innovation. Many breakthroughs in various fields have occurred through creative applications of mathematical concepts.

Cultural and Historical Significance: Mathematics has played a significant role throughout history and across cultures. Teaching mathematics also provides insight into the historical and cultural development of human knowledge.

In conclusion, mathematics is more than just a subject to be studied in school; it is a fundamental skill that equips individuals with the tools and knowledge they need to succeed in an increasingly complex and data-driven world. It fosters critical thinking, problem-solving, and logical reasoning, which are valuable skills for life and careers. Therefore, it is essential that mathematics remains a core part of the school curriculum.

Self-Check Exercise - 4

Discuss the place of mathematics in school curriculum.

1.7 Summary

In this unit, we studied the meaning, nature, importance and place of Mathematics in school curriculum. Definitions of mathematics given by different mathematicians/philosophers have been discussed. We found that “Mathematics is a way of thinking, an art and a human achievement.**”** We have also discussed the nature of mathematics in the following contexts.

- Mathematics is a Science of Discovery
- Mathematics is an intellectual game
- Mathematics is the art of drawing Conclusions

- Mathematics is a tool subject
- Mathematics is a system of logical processes
- Mathematics is an intuitive method

Mathematics is so important and far-ranging a subject of study that the argument for its compulsory inclusion in the secondary curriculum may be made from a variety of viewpoints. It should necessarily be studied throughout the secondary school for many reasons: It is beautiful, develops the mind, underpins the study of other subjects, is a necessary component of many jobs and is necessary to be a successful citizen; it is historically of great significance; and, finally, it is unique amongst human intellectual development.

1.8 Glossary

Whole Number: A whole number is a positive integer.

Finite: Not infinite; has an end.

Formula: A rule that numerically describes the relationship between two or more variables.

1.9 Answers to Self-Check Exercises

Self-Check Exercises- 1

1. d) All of the above

2. b) Reasoning

Self-Check Exercises- 2

1. Nature of Mathematics;

- Mathematics is a Science of Discovery
- Mathematics is an intellectual game
- Mathematics is the art of drawing Conclusions
- Mathematics is a tool subject
- Mathematics is a system of logical processes
- Mathematics is an intuitive method

2. d) All of the above
3. c) Logical

Self-Check Exercises -3

Answer: “Mathematics is the mirror of civilization”. Mathematics helps in the preservation, promotion and transmission of cultures. Concepts like symmetry, order, similarity, form and size form the basis of all works of art and beauty. All poetry and music utilizes mathematics. Hence, the teaching of mathematics is inevitable in our schools may be at any level. There are uses of mathematics in all the sciences and

arts, such as sculpture, drawing, and music. Mathematical skills are crucial for a wide array of analytical, technological, scientific, security and economic applications. To understand many institutions and their management problem, a quantitative viewpoint (modeling) is necessary. Many vocations need mathematical skills.

Self-Check Exercise - 4

Answer: Mathematics is an essential and integral part of the school curriculum for several important reasons like mathematics teaches students how to think critically, analyse problems, and develop problem-solving skills. Without a solid foundation in mathematics, students would struggle to comprehend and contribute to the advancements in fields such as physics, engineering, computer science, and more. Mathematics is more than just a subject to be studied in school; it is a fundamental skill that equips individuals with the tools and right knowledge. Therefore, it is essential that mathematics remains a core part of the school curriculum.

1.10 References/Suggested Readings

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1.11 Terminal Questions

1. Explain the meaning and nature of mathematics.
2. Discuss the importance of mathematics at school level education.

3. Discuss the place of mathematics in school curriculum.

Unit - 2

Aims and Objectives of Teaching Mathematics: Concept and Meaning, General Aims of Teaching Mathematics, Formulation and Classification of Objectives in Behavioural terms with Reference to Cognitive, Psycho-motor and Affective Domains

Structure

2.1 Introduction

Mathematics is a fundamental discipline that impacts various aspects of our everyday existence. Dear learner, this unit deals with the general aims and

objectives of teaching of mathematics in school curriculum. It also deals with formulation and classification of objectives in behavioral terms.

2.2 Learning Objectives

After completing this unit, you will be able to:

- explain the concept and general aims and objectives of teaching of mathematics.
- describe the process of formulation and classification of objectives in behavioural terms.

2.3 Concept and General Aims and Objectives of Teaching of Mathematics

Education is a process designed to bring about changes in individuals in specific ways, such as the development of interests, attitudes, and skills, to help them engage in certain activities. This process enables individuals to lead productive, socially acceptable lives. An objective defines the desired endpoint of action and reflects the purpose of the educational process. It represents the first step in teaching and learning, as it marks the starting point for activities, planning, and instruction. Objectives also provide the foundation for selecting evaluation procedures and guiding curriculum development, validating the entire teaching-learning process.

The term “objective” is defined as “a point or end view of a possible achievement, expressed in terms of what a student can do when the educational system is focused on its aims.” Therefore, an objective is a specific part of the broader educational aim that a school hopes to achieve. In this sense, an objective is narrower than an aim. It is a statement that describes what the student will be able to do in achieving an educational aim. When a student reaches an objective, they fulfill a part of the broader aim. Simply put, an objective describes the expected behavior at the end of a learning period. It is a form of statement indicating any kind of change, guiding the student's growth and providing a basis for evaluation procedures. Objectives create a connection between teachers, students, testers, and parents by focusing on the intended outcomes of learning, thereby validating the educational process.

It’s important to understand the difference between aims and objectives. Aims are general statements outlining the overall goals or intentions of teaching, while objectives are the specific stages learners must achieve to meet these goals. For example, a teacher may aim for a student to be able to save work on a computer. To achieve this aim, objectives such as creating a folder, navigating menus, and saving a document must be met. Aims and objectives often form hierarchical structures, where aims at one level may become objectives at another. Objectives focus on specific actions and exclude vague terms like “know” or “understand.” Instead, they

use active verbs like “state,” “explain,” “outline,” “list,” or “describe,” clearly specifying what learners are expected to do.

Objectives are categorized into two types:

1. **Educational Objectives**: These are broad and philosophical in nature, related to schools and educational systems. According to E.J. Frust, an educational objective is “a desired change in the behavior of a person that we try to bring about through education.” Bloom defined educational objectives as the goals that shape the curriculum and instruction, specifying the curriculum and evaluation techniques used. Educational objectives are accomplished with the help of teaching or instructional objectives.
2. **Teaching Objectives**: These are narrower and psychological in nature, focused on specific changes in behavior within a defined period, often within the classroom. They are related to the expected behavioral changes in the child and are called behavioral objectives. Teaching objectives are directly connected to the learning process and are clear, specific, measurable, and well-defined. They provide direction for the learning experience and teaching methods, forming the foundation of the educational structure. As such, teaching objectives are also known as instructional objectives. Teaching strategies, methods, and techniques are chosen based on these instructional objectives.

Aims and the objectives may be compared based on the following points:

Difference between Aims and Objectives

Sr.	**Aims**	**Objectives**
1.	Aims are very broad and comprehensive	Objectives are narrower and specific
2.	Philosophy and sociology areisthe main source**s** of aim**s**	Psychology is the main source of objectives
3.	These are not definite and clear	These are definite and clear
4.	These are difficult to achieve	These can be achieved
5.	Long duration is required for the achievement of aims	Achieved within a short duration of time i.e. within the classroom period
6.	These are subjective	These are objective
7.	These cannot be evaluated	These can be evaluated
8.	These include objectives	Objectives are a part of the aims
9.	They are related to the whole education system and the whole curriculum	These are related to the teaching of any specific topic
10.	It is the responsibility of the school, society and nation to achieve them	Generally teacher is responsible
11.	These are theoretical and indirect	Objectives are direct and concerned

		with the teaching-learning process
12.	Aims are formal	These are functional and informative

General Aims and Objectives of Mathematics Teaching

1. Utilitarian or Practical Aim:
The practical aims of teaching mathematics include:

- Helping students develop a clear understanding of the concept of numbers.
- Providing individuals with the knowledge of number-related ideas and operations required in everyday life.
- Enabling students to understand how numbers are applied in various measurements, particularly those used frequently, such as length, volume, area, weight, temperature, and speed.
- Ensuring that students become proficient in the four basic operations: addition, subtraction, multiplication, and division.
- Laying the foundation for mathematical skills and processes necessary for vocational purposes.
- Assisting learners in acquiring mathematical skills and attitudes to meet the demands of daily life, future mathematical work, and related fields of knowledge.
- Enabling students to make appropriate approximations.
- Helping learners understand ratios and scale drawings, as well as read and interpret graphs, diagrams, and tables.
- Equipping individuals with the ability to apply mathematics to solve a wide range of real-life problems.

2. Disciplinary Aim:
The teaching of mathematics aims to:

- Provide opportunities for learners to exercise and discipline their mental faculties.
- Encourage the intelligent use of reasoning power.
- Foster constructive imagination and inventive thinking.
- Develop systematic, orderly habits in students.
- Help learners become original and creative thinkers.
- Promote self-reliance and independence in students.

3. Cultural Aim:
The cultural aims of teaching mathematics include:

- Helping students appreciate the role of mathematics in past and present cultures.
- Teaching learners how mathematics preserves and transmits cultural traditions.

- Encouraging an appreciation of cultural arts such as drawing, painting, music, sculpture, and architecture through mathematical ideas.
- Providing intellectual and aesthetic enjoyment through mathematical concepts and promoting creative expression.
- Inspiring students to explore creative fields like art and architecture.
- Making students aware of the strengths and virtues of their inherited culture.
- Developing an aesthetic awareness of mathematical shapes and patterns in nature and human-made creations.

4. Social Aims:

The social aims of teaching mathematics focus on:

- Developing an understanding of mathematical principles and operations to enable active participation in the social and economic life of the community.
- Helping students understand how mathematical methods—scientific, intuitive, deductive, and inventive—are used to investigate, interpret, and make decisions in human affairs.
- Fostering the acquisition of social and moral values that contribute to a fruitful life in society.
- Supporting the formation of social laws and order needed for harmony within society.
- Providing scientific and technological knowledge essential for adjusting to a rapidly changing society.
- Encouraging students to appreciate how mathematics contributes to their understanding of natural phenomena.
- Helping students interpret social and economic phenomena.

General Aims of Teaching Mathematics:

- Foster an interest in mathematics.
- Cultivate a positive attitude towards learning mathematics.
- Develop speed and accuracy in mathematical calculations.
- Enable students to perform mathematical operations confidently.
- Encourage precise, logical, and critical thinking in problem-solving.
- Nurture investigative skills in mathematics.
- Teach students to recognize and apply mathematical relationships in everyday life.
- Enable students to analyze, synthesize, evaluate, and generalize to solve mathematical problems.
- Equip students to collect, organize, represent, analyze, and interpret data and make informed conclusions.
- Help students apply mathematical knowledge to both familiar and unfamiliar situations.
- Foster an appreciation of the role and value of mathematics in society.

- Encourage collaboration and teamwork in mathematical work.
- Prepare students with the necessary knowledge and skills for further education and training.
- Develop the ability to communicate mathematical ideas effectively.

Aims of Teaching and Learning Mathematics:

The overall aims of teaching and learning mathematics are to encourage students to:

- Recognize that mathematics is present in the world around us.
- Appreciate the usefulness, power, and beauty of mathematics.
- Enjoy mathematics and develop patience and persistence when solving problems.
- Understand and use the language, symbols, and notation of mathematics.
- Cultivate mathematical curiosity and use inductive and deductive reasoning when solving problems.
- Become confident in using mathematics to analyze and solve problems in school and real-life situations.
- Develop the knowledge, skills, and attitudes necessary for further studies in mathematics.
- Enhance abstract, logical, and critical thinking and the ability to reflect on their work and others' work.
- Appreciate the role of information and communication technology in mathematics.
- Understand and appreciate the international and multicultural dimensions of mathematics, as well as its historical development.

Objectives of the Secondary School Mathematics Education

Dr. Benjamin S. Bloom (1956) has classified the changes in behaviour into three categories or Domains:-

1. Cognitive Domain
2. Affective Domain
3. Psychomotor Domain

Dr. B.S. Bloom and his associates at the University of Chicago gave the classification of objectives of all three domains.

1. Classification of cognitive domain or objectives by Bloom (1956)
2. Affective domain by Krathwohl (1964)
3. Psychomotor Domain by Simpson (1986)

Dr. Bloom concentrated on the study of the cognitive domain. He assumed that in thinking about a problem a hierarchy of cognitive process is involved. While teaching, a teacher follows this hierarchical order. This classification of objectives is known as “Taxonomy of Educational Objectives” or “Bloom's Taxonomy” of objectives.

Taxonomy of Objectives in the Cognitive Domain

Bloom and his associates have classified the objectives related to the cognitive domain into six categories arranged from the lowest to the highest level:

1 **Knowledge: Lowest level**

- ❖ Acquisition of knowledge concerning specific facts, terminology, principles, theories

2. **Comprehension:**

- ❖ Based on knowledge. If there is no knowledge no comprehension takes place.
- ❖ Basic understanding of facts, ideas, principles, theories, etc

3. **Application:**

- ❖ Automatically involves both the above categories
- ❖ Leaner is required to make use of abstract ideas, principles in particular or concrete situations

4. **Analysis:**

- ❖ After realization of the above objectives, the learner is expected to acquire an analysis of the concept, elements, relationship
- ❖ Breaking down

5. **Synthesis:**

- ❖ Ability to combine the different components of an idea, concept, or principles to produce an integrated picture
- ❖ To combine
- ❖ May arrive at something new or originate some novel thing or idea after synthesizing all that is known to him earlier.

6. **Evaluation:**

- ❖ Ability to make proper value judgments about what has been acquired in the form of Knowledge understanding application analysis and synthesis.

- Highest level

- The learner is expected to make proper decisions about the quantitative and qualitative value of a particular idea

Taxonomy of Objectives in Affective Domain:

1. Receiving: Initial category of objectives belonging to the affective domain

- Awareness of stimuli
- willingness is created to attend to the stimuli
- efforts are made to control the attention

2. Responding:

- The learner must be made to respond
- Willingness to respond
- Satisfaction in response

3. Valuing:

- It depends upon both the former categories that is receiving and responding
- The learner is expected to imbibe a definite value pattern towards different ideas, events, and objects
- Development of typical value patterns

4. Organization:

- Organizing different value patterns imbibed by him from time to time
- Form a set value structure or Philosophy of life

5.Characterizing by a value or value complex:

- Highest level
- Include all the above categories
- Lifestyle of own
- The ultimate goal of the process of Education
- Known by own value system

Taxonomy of Objectives in the Psychomotor Domain:

1. Imitation:

- Impulsion

- ❖ Overt repetition

2. Manipulation:

- ❖ Following direction
- ❖ Selection
- ❖ Fixation

3. Precision:

- ❖ Reproduction
- ❖ Control

4. Articulation:

- ❖ Sequence
- ❖ Harmony

5. Naturalization

- ❖ Automatism
- ❖ Interiorization

Classification of Objectives of Secondary School Mathematics Education:

The classification of objectives of secondary school mathematics education is as under;

Knowledge Domain: To induce children to understand and grasp the knowledge of the following:

- The directed numbers and the real number system;
- The algebraic symbols to describe relations among quantities and number patterns;
- The equations, inequalities, identities, formulas and functions;
- The measures for simple 2-d and 3-d figures;
- The intuitive, deductive and analytic approach to study geometric figures;
- The trigonometric ratios and functions,
- The statistical methods and statistical measures;
- The simple ideas of probability and laws of probability

Skill Domain: To develop the following skills and capabilities in:

- Basic computations in real numbers and symbols and an ability to judge the reasonableness of results;

- Using the mathematical language to communicate ideas:
- Reasoning mathematically, i.e. They should conjecture, test, and build arguments about the validity of a proposition;
- Applying mathematical knowledge to solve a variety of problems;
- Handling data and generating information; number sense and spatial sense;
- Using modern technology appropriately to learn and do mathematics;
- Learning mathematics independently and collaboratively for the whole life

Attitude Domain: To foster the attitudes to:

- Be interested in learning mathematics;
- Be confident in their abilities to do mathematics;
- Willingly apply mathematical knowledge;
- Appreciate that mathematics is a dynamic field with its roots in many cultures;
- Appreciate the precise and aesthetic aspect**s** of mathematics;
- Appreciate the role of mathematics in human affairs;

List of Action Verbs used in formulating Learning Objectives

The following list contains examples of verbs which that describe the sorts of things you want your students to be able to do and may help you to write useful learning objectives.

Knowledge

Analyse	Arrange	Calculate	Circle	Cite
Classify	Compare	Contrast	Compare	Define
Describe	Match	Differentiate	Group	Identify
Interpret	Itemize	Label	List	Match
Name	Outline	Plan	Record	Revise
Select	Solve	State	Give examples	
Evaluate	Recognize			

Skills

Adjust	Assemble	Chart	Collect	use
Draw	Employ	establish	Illustrate	imitate
Interact	Locate	maintain	Measure	modify
Make	Organize	rearrange	Return	set up
Practice	Manipulate	Master	Fit	perform
demonstrate				

Attitudes

Accept	Adopt	advocate	approve	Assess
Challenge	Characterize	Choose	Criticize	Defend
Evaluate	Formulate	Judge	Justify	Manage
Model	Persuade	recommend	resolve	Select
Specify	Value	Reassure	empathies	

Objectives of Teaching Mathematics - National Policy of Education (1986)

At the end of the high school stage, students should be able to:

1. **Acquire Knowledge and Understanding**:
    - Gain a solid understanding of terms, concepts, principles, processes, and symbols used in mathematics.
    - Master fundamental mathematical processes, including computation, that are essential for daily life and further learning in mathematics.
2. **Develop Skills in Drawing and Measuring**:
    - Build proficiency in skills such as drawing, measuring, estimating, and demonstrating mathematical ideas effectively.
3. **Apply Mathematical Knowledge**:
    - Use mathematical knowledge and skills to solve practical problems encountered in daily life.
    - Apply mathematics to problems related to higher learning in mathematics or other allied fields.

4. **Enhance Thinking and Reasoning Abilities**:
    - Cultivate the ability to think critically, reason logically, and analyze mathematical situations.
    - Develop the capacity to articulate ideas in a coherent and logical manner.
5. **Appreciate the Power and Beauty of Mathematics**:
    - Develop an appreciation for the elegance, power, and beauty inherent in mathematics, fostering a deeper connection to the subject.
6. **Engage with Mathematics Enthusiastically**:
    - Show a keen interest in mathematics by participating in mathematical competitions and engaging in various mathematical activities.
7. **Respect for Great Mathematicians**:
    - Cultivate respect for mathematicians, especially Indian mathematicians, recognizing their contributions to the advancement of mathematical knowledge.
8. **Acquire Technological Proficiency**:
    - Develop the skills necessary to work with modern technological tools such as calculators, computers, and other mathematical devices.

These objectives align with the goal of preparing students to use mathematics not only as an academic subject but as a tool for practical problem-solving, logical reasoning, an

Self-Check Exercise- 1

1. Discuss the aims and objectives of teaching of mathematics.

2. Which of the following is the domain of learning according to Bloom?

a) Professional

b) Experimental

c) Social

d) Affective

3. According to Bloom's taxonomy remembering is a factor of _____ objective

a) Effective

b) Psychomotor

c) Cognitive

d) Social

4. The ability to use learnt material in a new situation by a child making use of his previous knowledge to solve the problem is called

a) Analysis

b) Synthesis

c) Application

d) Knowledge

5. According to Bloom’s taxonomy of educational objectives, the lowest level of the cognitive domain is

a) Analysis

b) Evaluation

c) Comprehension

d) Knowledge

2.4 Formulation and Classification Writing of Objectives in Behavioural Terms

Four Essential Elements of Well-Written Behavioral Objectives

When writing behavioral objectives for the classroom, it’s important to ensure that they meet the following four criteria:

1. **Student-Oriented**:
   A well-written behavioral objective focuses on what the student is expected to do, not on what the teacher will do. The emphasis is placed on the student’s actions, such as "The student will be able to…" rather than what the teacher will demonstrate or teach. This makes the objective relevant to the learner's performance, guiding both the teacher and the student in understanding the end goal.
2. **Describes Learning Outcomes**:
   The objective should describe what the student will learn to do or the specific skills they will acquire. It’s not about the teaching activities but the result of those activities — the learning outcomes. For example, instead of stating "The teacher will demonstrate multiplication," the objective should focus on what the student will achieve, such as "The student will multiply two-digit numbers accurately."
3. **Clear and Understandable**:
   A good behavioral objective uses simple, straightforward language and contains a clear verb that defines the expected behavior. The verb should indicate a specific, measurable action, such as "define," "calculate," or "demonstrate." The objective should be easily understood by both the teacher and the student, ensuring clarity about what is expected.
4. **Observable**:
   Learning outcomes must be observable so that assessment can take place. The action in the objective must be something that can be directly seen, heard, or measured. For instance, verbs like "write," "identify," "draw," and

"solve" are observable, as they result in clear, measurable actions or products. An objective that uses non-observable verbs, such as "understand" or "know," is difficult to evaluate. Observable outcomes allow teachers to assess whether the student has achieved the objective.

By ensuring that these four elements are part of your behavioral objectives, you can create clear, measurable, and effective goals that guide both teaching and learning.

Characteristics/Guidelines for Writing Behavioural Objectives

- Observable and measurable
- Results oriented/clearly written /specific
- Measurable by both quantitative and qualitative criteria
- Communicate successful learning in behavioural terms
- Written in terms of performance
- Begin each objective with an “action verb” which depicts definite, observable behaviour. For example: “write”, “identify”, “formulate”, “list”, “describe”, “recall”.
- State each objective in terms of student performance rather than teacher performance.
- State each objective as a learning product (outcome or terminal behaviour) rather than in terms of the learning process.
- State only one outcome or behaviour in each objective.
- Make objectives clear, brief and unambiguous.
- Start a set of behavioural objectives for a lesson with a phrase such as: “At the end of the lesson the student will be able to:”
- Do not include trivial objectives.
- Objectives can be written in present or future tense.
- Objectives will include four4 distinct components: Audience, Behaviour, Condition and Degree.
- Objectives must be both observable and measurable to be effective.
- Use of words like know, understand and learn in writing objectives are generally not acceptable as they are difficult to measure.
- Written objectives are a vital part of instructional design because they provide the roadmap for designing and delivering the curriculum.

- Throughout the design and development of curriculum, a comparison of the content to be delivered should be made to the objectives identified for the program. This process, called performance agreement, ensures that the final product meets the overall goal of instruction identified in the first level objectives.

Approaches of Writing Objectives in Behavioural Terms:

1. Robert Mager's Approach: Instructional objectives are best described in terms of terminal behaviour expected from the learner. He adopted Bloom's taxonomy and gave verbs for cognitive and affective domains.

2. Robert Miller's Approach: Mager approach badly neglected the psychomotor domain. Miller put his scheme based on skill analysis

3. RCEM Approach: (Regional College of Education Mysore)

Regional College of Education, Mysore has developed an approach for writing objectives in behavioral terms. It is applicable for cognitive, affective, and psychomotor objectives. There are four objectives which are used in this approach are –: (I) Knowledge (II) Understanding (III) Application and (IV) Creativity.

This approach uses mental processes in place of action verbs in writing instructional objectives.

KNOWLEDGE: Recall, Recognize

UNDERSTANDING: Seeing relationship, cite example, discriminate, classify, interpret, verify, and generalize

APPLICATION: Reason out, infer, predict, formulate hypothesis, establish hypothesis

CREATIVITY: Analyze, Synthesize, Evaluate

Essentials of Learning Objectives: Learning objectives form the basis for what is to be learned, how well it is to be performed, and under what conditions it is to be performed. While there are specific objectives that means different things, such as educational, instructional, learning behavioural, and performance objectives (Saettler, 1990); most instructional designers generally use two terms - terminal or performance objectives and enabling or learning objectives (Mager, 1975);

A Terminal or Performance Objective is developed for each of the tasks selected in the learning program. A terminal objective is at the highest level of

learning, appropriate to the human performance requirements a student will accomplish.

Each terminal performance objective is then analysed to determine if it needs one or more Enabling or Learning Objectives. These supporting objectives allow the Terminal Objective to be broken down into smaller, more manageable objectives. Each enabling learning objective measures an element of the terminal performance objective. The Three Parts of an Objective: Every performance or learning objective contains at least three parts:

- Observable Action (task): This describes the observable performance or behaviour. An action means a verb must be in the statement, for example "type a letter" or "lift a load." Each objective should cover one behaviour, hence, normally only one verb should be present. If there is more than one behaviour or the behaviour is complicated, then the objective should be broken down into one or more enabling learning objectives that support the main terminal learning objective.
- At Least One Measurable Criterion (standard): A criterion states the level of acceptable performance of the task in terms of quantity, quality, time limitations, etc. This will answer any question such as “How many? “or “How fast?” or “How well?” For example, “At least 5 will be produced”, “Within 10 minutes”, and “Without error.” There can be more than one measurable criterion. Do not fall into the trap of putting in a time constraint because you think there should be a time limit or you cannot easily find another measurable criterion- use a time limit only if required under normal working standards.
- Conditions of performance: A condition describes the actual conditions, **in** which the task will occur or be observed. In addition, it identifies the tools, procedures, materials, aids, or facilities to be used in performing the task. This is best expressed with a prepositional phase such as ‘without reference to a manual” or “by checking a chart.”

Examples of Performance Objectives

Example 1: Write the definition of rational numbers with 100% accuracy on their notebooks

Observable Action: Write the definition of rational numbers

Measurable Criteria: with 100 % accuracy

Conditions of Performance: on their notebooks

Four Parts of an ABCD Objective

- Audience
- Behaviour

- Condition
- Degree

The objective does not have to be written in this order (ABCD), but it should contain all of these elements

Audience

- Example: The 8th grade student.
- Example: The 6th class student.

Behavior

- Example... should be able to write a report...
- Example...should be able to describe the steps...

Condition

- Example: ...on their notebooks
- Example:...orally in front of class

Degree

- Example:... without error.
- Example:... 5 out of 10.
- Example:...within 30 seconds.

Example: The ninth-grade student will be able to recall the definition of "prime numbers" on a short-answer test and write it with 90% accuracy.

- Audience: Ninth-grade student
- Behaviour: Will be able to recall and write
- Conditions: On a short-answer test
- Degree: With 90% accuracy

Self-Check Exercise - 2

Describe three essential conditions of writing objectives in behavioural terms.

2.5 Summary

In this unit, general aims, objectives of teaching of mathematics and formulation and classification of objectives in behavioural terms are discussed in detail.

2.6 Glossary

Axiom: a proposition that is not actually proved or demonstrated, but is considered to be self-evident and universally accepted as a starting point for deducing and inferring other truths and theorems, without any need of proof.

Taxonomy - a classification of organisms based on similarities.

Blooms Taxonomy - A classification of educational objectives developed in the 1950s by a group of researchers headed by Benjamin Bloom of the University of Chicago. The taxonomy comprises three learning domains - cognitive, affective and psychomotor.

Knowledge - the result of perception, learning, and reasoning.

Synthesis - the combination of ideas into a complex whole.

2.7 Answers to Self-Check Exercise

Self-Check Exercise-1

1. Answer: An objective presents the endpoint towards which action is directed and therefore it reflects the purposefulness of the educational process. It represents the first step in the teaching and learning process because it is the starting point of activities, planning and instruction. Thus, an objective is a part of the aim which a school can hope to achieve. Objectives provide a link between teachers, pupils, testers and parents by focusing their attention on the intended outcomes of learning. The aims of teaching and learning mathematics are to encourage and enable students to understand and be able to use the language, symbols and notation of mathematics. It develops the knowledge, skills and attitudes necessary to pursue further studies in mathematics

2. d) Affective

3. c) Cognitive

4. c) Application

5. d) Knowledge

Self-Check Exercise- 2

Answer: Behavioural objectives that are useful in the classroom must meet certain criteria. Good behavioural objectives must be student-oriented**.** Good behavioural objectives must describe learning outcomes. Behavioural objectives must be clear and understandable. It should contain a clearly stated verb that describes a definite action or behaviour and, in most cases, should refer to an object of that action. Every performance or learning objective contains at least three parts: Observable Action (task), at Least One Measurable Criterion (standard) and Conditions of performance.

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2.9 Terminal Questions

1. What are the aims of objective of teaching Mathematics at Secondary state?
2. Describe any four characteristics of behavioral objectives

Unit - 3

Concept of Vedic Mathematics, Contributions of Indian Mathematicians: Bhaskaracharya, Aryabhatta and Ramanujam; Contributions of Western Mathematicians: John Venn and Pythagoras

Structure

3.1 Introduction

Dear learner, this unit deals with the concept of Vedic Mathematics, Contributions of Indian Mathematicians: Bhaskaracharya, Aryabhatta and Ramanujam; Contributions of Western Mathematicians: John Venn and Pythagoras in the field of mathematics.

3.2 Learning Objectives

After completing this unit, you will be able to:

- explain the concept of Vedic Mathematics.
- discuss the contributions of Indian Mathematicians: Bhaskaracharya, Aryabhatta and Ramanujam.
- discuss the contributions of Western Mathematicians: John Venn and Pythagoras in the field of mathematics

3.3 Concept of Vedic Mathematics

Vedic Mathematics, as rediscovered by Sri Bharati Krishna Tirthaji between 1911 and 1918, is a fascinating system of mathematics that draws upon ancient Indian

texts known as the Vedas. This system is based on sixteen *Sutras*, or word-formulas, which Tirthaji claimed were the foundation of all mathematical operations. The simplicity, coherence, and versatility of these Sutras allow for quick mental calculations and problem-solving, even for complex problems. The Vedic system has garnered growing interest in education due to its effectiveness in simplifying mathematics and fostering creativity in students.

Features of Vedic Mathematics:

1. **Speed and Simplicity:**
    - Vedic Mathematics allows for rapid mental calculations, even for problems that would typically require lengthy or complex methods in conventional systems.
    - Many methods, such as multiplication and division, can be performed with ease using simple steps that are easy to memorize and execute mentally.
2. **Unified System:**
    - One of the most striking aspects of Vedic Mathematics is its coherence. The various techniques and Sutras are interconnected, and solutions to problems can often be found by simply reversing a method.
    - For example, the same general multiplication technique can also be applied to division, and the squaring method can be reversed to calculate square roots.
3. **Flexibility and Creativity:**
    - The Vedic system allows students to develop their own approaches to problem-solving. This flexibility promotes creativity and critical thinking, encouraging students to explore mathematical relationships in their own way.
    - There is no single "correct" method; students are free to experiment and discover alternative solutions.
4. **Mental Calculations:**
    - The system is designed for mental calculations, which makes it particularly advantageous for students. Although the methods can be written down, the focus is on developing the ability to solve problems without relying on paper and pen.
    - This contributes to faster thinking and enhances the ability to process information in the mind.
5. **Encourages Mathematical Interest:**
    - The simplicity and beauty of the system can spark greater interest in mathematics among students, as the methods are engaging and intuitive. This leads to a deeper understanding of mathematical concepts.
6. **Holistic Approach:**

- Vedic Mathematics offers a comprehensive and systematic approach to mathematics, encompassing areas such as arithmetic, algebra, geometry, and calculus.
- The system's interconnected nature means that mastering one method often leads to an easier understanding of other areas.

7. **Educational Research:**
   - Interest in Vedic Mathematics is growing in educational circles. Research is being conducted to study the impact of learning Vedic Mathematics on children's cognitive development and their ability to solve problems.
   - Additionally, educators are exploring the integration of Vedic Sutras into modern mathematical fields, such as geometry, calculus, and computing.

Criticism and Skepticism:

Despite its popularity and many supporters, the Vedic system has also faced criticism and skepticism. One of the major criticisms is that the Sutras described by Tirthaji are not found in the standard editions of the Vedic texts. Tirthaji himself claimed that the sixteen Sutras were not present in the standard editions but were part of a different version of the Atharvaveda, which was not available to scholars outside India at the time.

Some critics argue that Tirthaji's attribution of the mathematics to the Vedas may be more of a symbolic or interpretive claim rather than a direct historical connection to the texts themselves. There is also skepticism about the accuracy of Tirthaji's translations and interpretations of the Sanskrit language used in the Vedas.

Conclusion:

Despite these criticisms, the practical applications and advantages of Vedic Mathematics are undeniable. The system offers a faster, more creative, and engaging approach to learning and solving mathematical problems. It provides a valuable alternative to conventional methods, especially in educational settings, where students can benefit from the flexibility, speed, and mental agility promoted by the Vedic approach. While some aspects of the historical background may be contentious, the practical benefits of Vedic Mathematics continue to inspire students, teachers, and mathematicians worldwide.

The 16 sutras are as follows:

Sr.	Name	Corollary	Meaning

1.	Ekadhikena Purvena	Anurupyena	By one more than the previous one
2.	Nikhilam Navatashcaramam Dashatah	Sisyate Sesasamjnah	All from 9 and the last from 10
3.	Urdhva-Tiryagbyham	Adyamadyenantyamantyena	Vertically and crosswise
4.	Paraavartya Yojayet	Kevalaih Saptakam Gunyat	Transpose and adjust
5.	Shunyam Saamyasamuccaye	Vestanam	When the sum is the same that sum is zero
6.	Anurupye Shunyamanyat	Yavadunam Tavadunam	If one is in ratio, the other is zero
7.	Sankalana-vyavakalanabhyam	Yavadunam Tavadunikritya Varga Yojayet	By addition and by subtraction
8.	Puranapuranabyham	Antyayordashake'pi	By the completion or non-completion
9.	Chalana-Kalanabyham	Antyayoreva	Differences and Similarities
10.	Yaavadunam	Samuccayagunitah	Whatever the extent of its deficiency
11.	Vyashtisamanstih	Lopanasthapanabhyam	Part and Whole
12.	Shesanyankena Charamena	Vilokanam	The remainder by the last digit
13.	Sopaantyadvayamantyam	Gunitasamuccayah Samuccayagunitah	The ultimate and twice the penultimate
14.	Ekanyunena Purvena	Dhvajanka	By one less than the previous one
15.	Gunitasamuchyah	Dwandwa Yoga	The product of the sum is equal to the sum of the product
16.	Gunakasamuchyah	Adyam Antyam Madhyam	The factors of the sum is equal to the sum of the factors

In Vedic mathematics, two concepts are the pillars of Mathematics. They are Base and Complements. **What is a Base?** Bases are the numbers which start with 1 and end with 0's. For example 10, 100, 1000, 10000, 100000, and so on are called bases. The first 2-digit number is 10 and the first 3-digit number is 100 and so on. Only these kinds of numbers are considered as bases. Numbers such as 200, 300, 450, 1500 are not considered as bases. **What is a Complement?** When two numbers are added with to each other and it results in the next nearest base, then they are called Complements of each other. For Example, --- Consider the number 73. The next nearest base of 73 is 100. By adding 27 we will get 100. So 73 and 27 are complements to each other. Consider the number 156. The next nearest base of 156 is 1000. By adding 844 we get 1000. So they are Complements. Similarly, Complement of 6 is 4 (Base is 10).Complement of 25 is 75 (Base is 100).Complement of 6545 is 3455 (Base is 10000).How to find the Complement of a number in an easier way? Complement of a number can be calculated by **subtracting** all numbers from 9 and the last number by 10.e.g.: Complement of 358732? Solution: 9 9 9 9 9 10 – 3 5 8 7 3 2 = 6 4 1 3 6 8, The Formula is "**All from 9 and the last from 10"**.Complement of 5183?Solution :9 9 9 10 – 5 1 8 3 = 4 8 1 7, **What if the last digit is "Zero"??**In that cases treat the last non-zero number as the last digit and use the same above formula. E.g.: Complement of 69560670??

Solution: 9 9 9 9 9 9 10
– 6 9 5 6 0 6 7 0
3 0 4 3 9 3 3 0

Self-Check Exercise 1
What is complement?

3.4 Contributions of Indian Mathematicians (Bhaskaracharya, Aryabhatta and Ramanujam)

Bhaskara: Bhaskara or Bhaskaracharya is the most well-known ancient Indian mathematician. He was born on 1114A.D. at Bijjada Bida in the Sahyadari hill. He is famous for his book Siddhanta Shiromani (1150 A.D). It is divided into four sections- Leelavati (a book on arithmetic), Bijaganita(algebra), Goladnayaya (chapter on sphere- celestial globe), and Granoganita (mathematics of a planets), Leelavagti contains many interesting problems and was a very popular text book. He gave an example idea of what is now called - Rolle's Theorem. Unfortunately, later Indian mathematics did not take any notice of this. Five centuries, later Newton and Leibnitz developed these subjects. As an astronomer, Bhaskara is renowned for his concept of Tatakalikagati. Bhaskaracharya's work in Algebra, Arithmetic and Geometry catapulted him to fame and immortality. His renowned mathematical works called Lilavati and Bijaganita are considered to be unparalleled and a memorial to his profound intelligence. Its translation in several languages of the world bears

testimony to its eminence. In his treatise Siddant Shiromani, he writes on planetary positions, eclipses, cosmography, mathematical techniques and astronomical equipment. In the Surya Siddant, he makes a note on the force of gravity: “Objects fall on earth due to a force of attraction by the earth. Therefore, the earth, planets, constellations, moon, and sun are held in orbit due to this attraction.” Bhaskaracharya was the first to discover gravity, 500 years before Sir Isaac Newton. He was the champion among mathematicians of ancient and medieval India.

Bhaskaracharya wrote Siddhanta Shiromani in 1150 AD when he was 36 years old. This is a mammoth work containing about 1450 verses. It is divided into four parts, Lilawati, Beejaganit, Ganitadhyaya and Goladhyaya. In fact, each part can be considered as separate book. The number of verses in each part is as follows, Lilawati has 278, Beejaganit has 213, Ganitadhyaya has 451 and Goladhyaya has 501verses.

One of the most important characteristics of Siddhanta Shiromani is, it consists of simple methods of calculations from Arithmetic to Astronomy. Essential knowledge of ancient Indian Astronomy can be acquired by reading only this book. Siddhanta Shiromani has surpassed all the ancient books on astronomy in India. After Bhaskaracharya nobody could write excellent books on mathematics and astronomy in lucid language in India. In India, Siddhanta's works used to give no proof of any theorem. Bhaskaracharya has also followed the same tradition. Lilawati is an excellent example of how a difficult subject like mathematics can be written in poetic language. Lilawati has been translated in many languages throughout the world. When British Empire became paramount in India, they established three universities in 1857, at Bombay, Calcutta and Madras. Till then, for about 700 years, mathematics was taught in India from Bhaskaracharya’s Lilawati and Beejaganit. No other textbook has enjoyed suchalong lifespan. Bhaskaracharya has given the terms for numbers in multiples of ten and he says that these terms were coined by ancients for the sake of positional values. Bhaskar's terms for numbers are as follows:

eka(1), dasha(10), shata(100), sahastra(1000), ayuta(10,000), laksha(100,000), prayuta (1,000,000=million), koti(107), arbuda(108), abja(109=billion), kharva (1010), nikharva (1011), mahapadma (1012=trillion), shanku(1013), jaladhi(1014), antya(1015=quadrillion), Madhya (1016) and parardha(1017).

Bhaskara's arithmetic text Lilavati covers the topics of definitions, arithmetical terms, interest computation, arithmetical and geometrical progressions, plane geometry, solid geometry, the shadow of the gnomon, methods to solve indeterminate equations, and combinations. Lilavati is divided into 13 chapters and covers many branches of mathematics, arithmetic, algebra, geometry, and a little trigonometry and mensuration. His Bijaganita (“Algebra”) was a work in twelve chapters. It was the first text to recognize that a positive number has two square roots (a positive and a

negative square root). Using an astronomical model developed by Brahmagupta in the 7th century, Bhaskara accurately defined many astronomical quantities, including, for example, the length of the sidereal year, the time that is required for the Earth to orbit the Sun, as 365.2588 days[citation needed] which is same as in Suryasiddhanta. The modern accepted measurement is 365.2563 days, a difference of just 3.5 minutes. His mathematical astronomy text Siddhanta Shiromani is written in two parts: the first part on mathematical astronomy and the second part on the sphere. The twelve chapters of the first part cover topics such as: mean longitudes of the planets, true longitudes of the planets, the three problems of diurnal rotation, syzygies, lunar eclipses, solar eclipses, latitudes of the planets, sunrise equation, the moon's crescent, conjunctions of the planets with each other, conjunctions of the planets with the fixed stars and the patas of the sun and moon.

Summary of Bhaskara's Contributions

- A proof of the Pythagorean Theorem by calculating the same area in two different ways and then cancelling out terms to get $a^2 + b^2 = c^2$.
- In Lilavati, solutions of quadratic, cubic and quartic indeterminate equations.
- Solutions of indeterminate quadratic equations (of the type $ax^2 + b = y^2$).
- Integer solutions of linear and quadratic indeterminate equations (Kuttaka). The rules he gives are (in effect) the same as those given by the Renaissance European mathematicians of the 17th century
- A cyclic Chakravala method for solving indeterminate equations of the form $ax^2 + bx + c = y$. The solution to this equation was traditionally attributed to William Brouncker in 1657, though his method was more difficult than the chakravala method.
- His method for finding the solutions of the problem $x^2 - ny^2 = 1$ (so-called "Pell's equation") is of considerable interest and importance.
- Solutions of Diophantine equations of the second order, such as $61x^2 + 1 = y^2$. This very equation was posed as a problem in 1657 by the French mathematician Pierre de Fermat, but its solution was unknown in Europe until the time of Euler in the 18th century.
- Solved quadratic equations with more than one unknown, and found negative and irrational solutions.
- Preliminary concept of mathematical analysis.

- Preliminary concept of infinitesimal calculus, along with notable contributions towards integral calculus.

- Conceived differential calculus, after discovering the derivative and differential coefficient.

- Stated Rolle's Theorem, a special case of one of the most important theorems in analysis, the mean value theorem. Traces of the general mean value theorem are also found in his works.

- Calculated the derivatives of trigonometric functions and formulae. (See the Calculus section below.)

- In Siddhanta Shiromani, Bhaskara developed spherical trigonometry along with a number of other trigonometric results.

Aryabhatta: The Indian mathematician and astronomer Aryabhatta (476 A.D) is well known for his work. He was born at Pataliputra near Patna in Bihar. His most famous book is known as - Aryabhatiya. In arithmetic, Algebra and place Geometry Aryabhatta suggested numerous rules. Aryabhatta published his first book, Aryabhatiya, when he was 23. In it, he not only wrote about advanced ideas in mathematics and astronomy, but healsowrote in the tight poetry meters of the Hindu philosophical texts. Aryabhatta enhanced our understanding of the Earth and its place in the galaxy.He also wrote analyses on sundial measurements so that time could be measured more accurately. Aryabhatta gave an accurate approximation for π. He wrote in Aryabhatiya that, "Add four to one hundred, multiply by eight and then add sixty-two thousand."The result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given. This gives $\pi = {}^{62832}/_{20000} = 3.1416$ which is a surprisingly accurate value. In fact π = 3.14159265 correct to 8 places. A few important rules are listed below-

- Area of triangle = ½ *base*height
- The value of Pi =3.1456
- Area of the circle =πr^2
- Sum of AP =n/2[2a+ (n-1) d]

- A verse mentions that Aryabhata was the head of an institution (kulapa) at Kusumapura. Since, the University of Nalanda was in Pataliputra, and had an astronomical observatory; it is probable that he was its head too.
- Direct details of his work are known only from the Aryabhatiya. His disciple Bhaskara I calls it Ashmakatantra (or the treatise from the Ashmaka).
- The Aryabhatiya is also occasionally referred to as Arya-shatas-aShTa (literally, Aryabhata's 108), because there are 108 verses in the text. It also has 13 introductory verses, and is divided into four pādas or chapters.

- Aryabhatiya's first chapter, Gitikapada, with its large units of time - kalpa, manvantra, and Yuga — introduces a different cosmology. The duration of the planetary revolutions during a mahayuga is given as 4.32 million years.
- Ganitapada, the second chapter of Aryabhatiya has 33 verses covering mensuration (kṣetra vyāvahāra), arithmetic and geometric progressions, gnomon or shadows (shanku-chhAyA), simple, quadratic, simultaneous, and indeterminate equations.
- Aryabhatiya's third chapter Kalakriyapada explains different units of time, a method for determining the positions of planets for a given day, and a seven-day week with names for the days of week.
- The last chapter of the Aryabhatiya, Golapada describes Geometric/trigonometric aspects of the celestial sphere, features of the ecliptic, celestial equator, shape of the earth, cause of day and night, and zodiacal signs on the horizon.
- He did not use a symbol for zero; its knowledge was implicit in his place-value system as a place holder for the powers of ten with null coefficients.
- He did not use the Brahmi numerals, and continued the Sanskritic tradition from Vedic times of using letters of the alphabet to denote numbers, expressing quantities in a mnemonic form.
- He worked on the approximation for pi thus - add four to 100, multiply by eight, and then add 62,000; the circumference of a circle with a diameter of 20,000 can be approached.
- It is speculated that Aryabhata used the word āsanna (approaching), to mean that not only is this an approximation, but that the value is incommensurable or irrational.
- In Ganitapada, he gives the area of a triangle as: "for a triangle, the result of a perpendicular with the half-side is the area". He discussed 'sine' by the name of ardha-jya or half-chord.
- Like other ancient Indian mathematicians, he too was interested in finding integer solutions to Diophantine equations with the form ax + by = c; he called it the kuṭṭaka (meaning breaking into pieces) method.
- His contribution to the study of Algebra is immense. In Aryabhatiya, Aryabhata provided elegant results for the summation of a series of squares and cubes through well tried formulae.
- His system of astronomy was called the audayaka system, in which days are reckoned from uday, dawn at lanka or "equator". His later writings, which apparently proposed the ardha-r AtrikA, or midnight model, are lost.
- He correctly believed that the earth rotates about its axis daily and that the apparent movement of the stars is a relative motion caused by the rotation of the earth, challenging the prevailing view.
- In Aryabhatiya, he writes that the 'setting and rising of planets' is a perception similar to that of someone in a boat going forward who sees an unmoving (object) going backward.

- He correctly asserted that the planets shine due to the reflection of sunlight and that the eclipses occur due to the shadows of the moon and earth, and are not caused by a demon called “Rahu”.
- He correctly deduced that the orbits of the planets are ellipses; this is another great discovery not credited to him but to Johannes Kepler (a German astronomer, born AD 1571)

Aryabhata’s work was of great influence in on the Indian astronomical tradition and influenced several neighboring cultures through translations. Some of his works are cited by Al-Khwarizmi, and in the 10th century by Al-Biruni. The Aryabhata Knowledge University (AKU), Patna, was established by the Government of Bihar in his honor for the development and management of educational infrastructure related to technical, medical, management and allied professional education. India’s first satellite Aryabhata is named in his honour. At the Aryabhata Research Institute of Observational Sciences (ARIOS) near Nainital, India, research in astronomy, astrophysics and atmospheric sciences is conducted. To sum up, Aryabhatta was really one of the greatest geniuses of his time in the field of mathematics and astronomy.

Ramanujan: Ramanujan was born in Brahmin family on December 22, 1887 at Erode madras. He got his school education at Kumba Koram. He won a scholarship in the matriculation examination. His teachers were very much impressed by his injected and special gifted abilities in mathematics. Hardy remarked: I had never seen anything the least like them before. A single look at them is enough to show that they could be written down by a mathematician of the highest class. His work threw light on divergent series. Hypergeometric series continued fraction, definite integrals. Partition functions, ecliptic functions the theory of numbers, fractional differentiation and highly composite numbers.He excelled in mathematics as a school student, and mastered a book on advanced trigonometry written by S. L. Loney by the time he was 13. While in his mid-teens, he was introduced to the book ‘A Synopsis of Elementary Results in Pure and Applied Mathematics’ which played an instrumental role in awakening his mathematical genius. By the time he was in his late-teens, he had already investigated the Bernoulli numbers and had calculated the Euler - Mascheroni constant up to 15 decimal places. He was, however, so consumed by mathematics that he was unable to focus on any other subject in college and thus could not complete his degree. After years of struggling, he was able to publish his first paper in the ‘Journal of the Indian Mathematical Society’ which helped him gain recognition. He moved to England and began working with the renowned mathematician G. H. Hardy. Their partnership, though productive, was short-lived as Ramanujan died of an illness at the age of just 32.Considered to be a mathematical genius, Srinivasa Ramanujan, was regarded at par with the likes of Leonhard Euler and Carl Jacobi. Along with Hardy, he studied the partition function P(n) extensively and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. Their work led to the

development of a new method for finding asymptotic formulae, called the circle method.

3.5 Contributions of Western Mathematicians (John Venn and Pythagoras)

John Venn: John Venn was a British mathematician and philosopher born on 4thAugust 1834 in Hull, Yorkshire. His mother died when he was only three years of age. His father, who was the rector of the Parish of Drypool, was from a distinguished family. Venn had descended from a long line of church evangelicals. His early schooling was done from High gate and Islington. He then went to Gonville and Caius College of Cambridge in 1853 where he got a degree in mathematics in 1857. He became a Fellow of the College; a title he kept for life. Venn was brought up in a very strict atmosphere at home. His father Henry had played a significant part in the Evangelical movement and he was also the secretary of the 'Society for Missions to Africa and the East'. Thus he was compelled to follow the family tradition. Venn became a priest in 1859 after being ordained as a deacon at Ely. He also went to a town as a curate. However, his thirst for knowledge and passion for mathematics encouraged him to do more than this. He therefore moved back to Cambridge to lecture in moral sciences. Venn was very good in the branch of mathematics we call 'logic'. He has three textbooks to his name; 'The Logic of Chance' which was published in 1866, 'Symbolic Logic' (1881) and The Principles of Empirical Logic (1889). The books dealt with frequency interpretation that is which is the frequency theory of probability. The first book had a great influence on in the theory of statistics and its development. 'Symbolic Logic' was the book that gave the introduction of the Venn diagrams.

Venn Diagrams: John Venn came up with Venn Diagrams in 1881. They were a representation of the relation between sets using circles within circles. For example, if we take three circles A, B and C which are all subsets of D. The sections which are overlapping represent similar properties of the subsets whereas the independent areas were the individual properties of the sets. Venn diagrams can be applied in various problems however they particularly aided in the Boolean logic (named after the mathematician George Boole). Venn was elected as member of the Royal Society in 1883. He wrote a book 'The Biographical History of Gonville and Cauis College' which was published in 1897. He married Susanna Carnegie Edmonstone in 1867 with whom he had one son John Archibald Venn who later entered the mathematical field. He is remembered for his immense contribution to logic. There is a building named after him at the University of Hull and a stained glass window in a hall in the 'Gonville and Caius College' remembering his work. John Venn died on 4th April 1923**.**

Pythagoras: Pythagoras of Samos is one of the most famous names in the history of mathematics and is recognized as the first true mathematician. Most of the information we have today on this legendary mathematician were was compiled centuries after he lived and thus many are considered to be unreliable. His early

biographies are written by authors who wanted to present him as some supernatural or god like figure. It is said that before the birth of Pythagoras, it was prophesized that his pregnant mother would give birth to a man supremely beautiful, wise, and beneficial to human kind. He was born on the Greek Island - Samos in the eastern Aegean. His birth date is estimated to be somewhere in 570 BC. His father Mnesarchus was a merchant and travelled a lot for business, Pythagoras also accompanied his father in on various expeditions. When Pythagoras was 18, he visited Miletus- an ancient Greek city on the western coast of Anatolia; where he met Thales– the first known Greek philosopher and scientist. By that time Thales was very old and is not believed to have taught Pythagoras a great deal. However, it was this meeting which triggered his interest in the science of mathematics and astronomy. Thales advised him to travel to Egypt and explore these avenues. Pythagoras is believed to have had a strong desire to learn and for this he had undertaken extensive travels. He was taught by a wide range of teachers and philosophers. He spent years in Egypt in search of all available knowledge and received wisdom from an Egyptian priest Oenuphis of Heliopolis. In around 530 BC Pythagoras settled in Croton- Italy, where he founded a philosophical and religious school that instantly attracted many followers. He established and headed a society called mathematikoi. The members of his society lived permanently together and followed strict rules. Pythagoras taught all the members of the society personally. It is due to the strict rules, secrecy and communal system of his school that there is not much known of Pythagoras's actual work or it is really hard to distinguish his work from that of his followers. Pythagoras has commonly been credited for discovering the Pythagorean Theorem of geometry. Though this theorem was previously utilized by Babylonians and Indians; it is widely believed that Pythagoras or his students were the first to construct its proof. Pythagoras believed that numbers had personalities like perfect or incomplete, masculine or feminine, beautiful or ugly. He also studied properties of numbers that would be familiar to mathematicians today like even and odd numbers. Pythagoras desired to stay out of politics, yet his society was always affected by politics. In 510 BC Croton attacked and defeated its neighbour Sybaris and there are certainly some suggestions that Pythagoras became involved in the dispute. Then in around 508 BC the Pythagorean Society at Croton was attacked by Cylon, a noble from Croton itself. Pythagoras escaped to Metapontium and most authors say he died there, some claiming that he committed suicide because of the attack on his Society. The evidence is unclear as to when and where the death of Pythagoras occurred but his society expanded rapidly after 500 BC and its contributions to mathematics are still recognized and respected.

Self-Check Exercise 2

1. What is the value of phi?
2. Who discovered the right triangle?
    a) Pythagoras
    b) Ancient Egyptians
    c) Miley Cyrus

d) Michelangelo

3. At which age did Ramanunjan pass away?
   a) 26
   b) 31
   c) 33
   d) 39

3.6 Summary

Dear learner, in this unit you have studied the concept of Vedic Mathematics, Contributions of Indian Mathematicians: Bhaskaracharya, Aryabhatta and Ramanujam; Contributions of Western Mathematicians: John Venn and Pythagoras in the field of mathematics

3.7 Glossary

NIKHILAM NAVATASHCARAMAM DASHATAH (all from 9 and last from 10): This sutra is mostly used in subtracting a number from the powers of 10.

URDHVA-TIRYAGBHYAM (vertically and crosswise): This sutra again is used for multiplications and the formula is of the form:
ab x cd = (a.c) (a.d + b.c) (b.d) where '.' refers to multiplication.

3.8 Answers to Self-Check Exercise

Self-Check Exercise 1

Answer: When two numbers are added with each other and it results in the next nearest base, then they are called Complements of each other.

Self-Check Exercise 2

1.Answer: The value of Pi = 3.1456.
2. a) Pythagoras
3. c) 33

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3.10 Terminal Questions

1. Describe the concept of Vedic mathematics.
2. Explain the contributions of Indian mathematicians (Bhaskaracharya, Aryabhatta and Ramanujam).
3. Describe the contributions of Western mathematicians (John Venn and Pythagoras).

Unit - 4

Curriculum Construction in Mathematics, Selection and Organization, Factors affecting Change in Mathematics Curriculum, Process of Evaluation of Mathematics Curriculum at School Level

Structure

4.1 Introduction

Dear learner, this unit deals with the concept and principles of curriculum construction in mathematics, selection and organization of content in mathematics curriculum.

4.2 Learning Objectives

After completing this unit, you will be able to:

- discuss the concept of curriculum construction and selection and organization of content in mathematics
- explain the factors affecting change in mathematics curriculum and process

4.3 Concept of Curriculum

Dear learner, first of all we must know the difference between syllabus and curriculum. A syllabus is a descriptive outline and summary of topics that are to be covered in an education or training course. The syllabus will usually provide specific information about the said training course and is often drafted by the governing body or by the instructor of the course. The syllabus also serves as a means for the students to be aware and understand what they will be taught in the duration of the course. Syllabi, on the other hand, is the plural form of a syllabus. According to Dictionary.com, a syllabus is:

- An outline or other brief statement of the main points of a discourse, the subjects of a course of lectures, the contents of a curriculum, etc.
- A short summary of the legal basis of a court's decision appears at the beginning of a reported case.
- A book containing summaries of the leading cases in a legal field, used especially by students.

A curriculum is the set of courses, and their content, offered at a school or university. The term, 'curriculum' is derived from the Latin word “Currere” which means to run/to proceed. Currere refers to the 'course of deeds and experiences through which children grow to become mature adults. Dictionary.com defines curriculum as:

- The aggregate of courses of study given in a school, college, university, etc.
- The regular or a particular course of study in a school, college, etc.

The main difference between a syllabus and a curriculum is that a curriculum is a more generalized or an overview of the subjects or topics that the students are meant to learn. However, a syllabus is a more detailed overview of the subject of study. For example a mathematics curriculum may list the basics of algebra, the basics of geometry, and the basics of trigonometry, while the class syllabus will list what topics will be covered under each of the basic topics, what will be the concepts that students may understand by the end of each topic, and it may even list what exercises or problems in the textbook will be covered during class. Hence, it can be said that the syllabus is a subset of the curriculum.

Meaning and Definitions of Curriculum

The term curriculum derives from the Latin word 'currere' which means a kind of route that the learner travels. All the activities going on in the school or outside of the school are called curriculum. It is basic to the intellectual, physical, moral and emotional development of the child.

Secondary Education Commission (1952-53), "Curriculum is the totality of experiences that pupils receive through the manifold activities that go in the school, in the classroom, library, laboratory, workshop, playground and in the numerous informal contacts between the teachers and pupils".

Cunningham, "Curriculum is a tool in the hands of the artist (teacher), to mould his/her materials (students), according to his/her ideals (aims and objectives) in his/her studio (college/school)".

John Kerr, "All the learning which is planned and guided by the school, whether it is carried on in groups or individually, inside or outside the school".

Braslavsky states that "curriculum is an agreement amongst communities, education and the State on what learners should take on during specific periods of their lives."

A curriculum is a set of courses, including their content, offered at a school or university. The curriculum often contains a detailed list of subjects and the elements of teaching them. John Franklin Bobbitt's "The Curriculum" published in 1918 mentions curriculum as an idea that has its roots in the Latin word 'race-course'. He also explained "The Curriculum" as the course of deeds and experiences through which children grow up into adults and get going for success in society. A curriculum is more than putting together a set of academically required subjects. It must consider all aspects of the student life, the learning needs of students, the time available for the sessions and the teachers' ideas, capability and workload.

Major Defects in the Present Curriculum

- It is examination oriented (examination ridden)
- Not in conformity with the aims and objective of teaching mathematics
- Textbook based examinations
- Emphasis on theory not practical
- Heavy syllabus
- Rote learning is encouraged
- Not too life oriented
- Not helpful to vocation
- Not developing the whole personality

Principles of Curriculum Construction

The principles of curriculum construction are listed as under:

Principle of Child-Centeredness: The child should be the central figure in any scheme of curriculum construction. In fact, the curriculum is meant to bring about the development of the child in the desired direction so that he is able to adjust well in life. The curriculum is mainly meant for the students, so it should be child centred not teacher-centred. The age, ability, interest, capability, capacity, needs and psychology of the learner should be taken into consideration while developing/constructing the curriculum.

Principle of Community-Centeredness: After getting the education in the school, he has to live in and for the society. Therefore, his needs and desires must conform to the needs and desires of the society in which he is living. The values, attitudes and systems that are prevailing in the community must be reflected in the curriculum. In other words, the social needs of society and the needs of the learner should be

taken into account while constructing the curriculum. It should reflect the values of democracy, ethos and main concerns of the country.

Principle of Activity-Centeredness: The curriculum should centre around the multifarious activities of pupils. It should provide well-selected activities according to the needs, interests and developmental stages of children. It should provide constructive, creative and project activities. The purposeful activities both in the class-room and outside the class-room should be provided. It is through a network of activities that the desired experiences can be provided and consequently, desirable behavioural changes can be brought about in children

Principle of Utility: The main purpose of education is to prepare the child for living and learning. It is one of the most important considerations so that the child can live a self-fulfilling life. It will be possible when the curriculum should provide rich experiences (academic and social) to the students. The content, activities and experience of the curriculum at a particular stage / grade are useful to the learner for the further/higher studies.

Principle of Variety: The school curriculum should be broad-based so that it can accommodate the needs of different category children. The needs of pupils also change from place to place. For example, the pupils studying in rural, urban and hilly areas will have different needs. All these considerations should be reflected in the curriculum.

Principle of Integration: The study of different subjects and activities have to serve the same purpose i.e., to increase the achievement of learners and fulfill the aims of education. The activities and subjects should not be put in after-tight compartments but these should be interrelated and well integrated so as to develop the personality of child. The curriculum should integrate; cognitive, affective and psychomotor objectives and abilities, knowledge and experience, objectives and content, and child's activity. It should also be related to the social environment of the students.

Principle of Forward-Looking: The main aim of education is to enable the child to lead a healthy and good social life. The aim of the curriculum should not only be to cater present needs of the child alone. But the future needs of his life should be taken into consideration. The curriculum should include skills, experiences, influences etc. which will develop in the child abilities and power to make effective adjustments in the later life.

Principle of Flexibility: In social milieu, rapid developments are taking place everywhere almost in various fields. Therefore the needs of society are also changing with time and development. In this context, the content of curriculum cannot be the same for all times to come. It should not be static but dynamic. It must be changed with the changing needs and time. It should reflect the latest trends evolving in the field of education and psychology.

Principle of Creativity: It should place the pupil in the place of the discoverer and proper provision should be made for such type of activities. Principle of

Preservation/Conservation: It should help in the preservation /conservation and transmitting the knowledge, traditions, and standards of conduct on which the culture and civilization depend.

Principle of Contemporary Knowledge: The modern or current knowledge and theories should be taught to the students. That will give the knowledge of utilization of local resources (salt, plants, soil) to the students.

Principle of Conservation of our Cultural Heritage: Education is considered as a means for conserving our cultural heritage. Protection and transmission of culture are important components in this process. Mathematics curriculum needs to follow this principle and wherever possible, we need to include some activities, which would help in preserving and transmitting the cultural heritage. It is an important aspect that needs to be taken care of while framing the curriculum.

Self-Check Exercise - 1

1. What is Curriculum?
2. Which of the following statements is not related to the curriculum?
    a) It is related to the learning experience of the students.
    b) It is related to the program of study.
    c) It is related to the professional development of teachers.
    d) It is the structure of the curriculum framework.

3. Which of the following is not a guiding principle for curriculum development
    a) Principle of child centeredness
    b) Principle of Community-Centeredness
    c) Principle of teacher-centredness
    d) Principle of Activity-Centeredness

Self-Check Exercise - 2

Discuss any two principles of the curriculum.

4.4 Factors Affecting Change in Mathematics Curriculum

The education system of a country is affected by some particular factors, i.e. the National identity, historical and cultural backgrounds, and political, economic, social, scientific, and administrative factors. The impacts of all these factors show certain directions to the education system of the country. These directions can be useful in making newer policies and regulations for education according to the vision of the nation and its abilities on the whole.

1. **Social Factors**: The impact of social factors on education cannot be underestimated. From the influence of family dynamics and socioeconomic status to the role of peer relationships and cultural norms, these factors shape the educational experiences and outcomes of individuals. By recognizing the

profound effects of social factors and implementing strategies to address them, we can create a more inclusive and equitable educational landscape that empowers every learner to reach their full potential. The needs and requirements of society are dynamic. The changes in values, beliefs, and expectations can prompt curriculum changes. For example, the increasing importance of technology in our daily lives has led to a greater emphasis on Science, Technology, Engineering, and Mathematics education in school curriculum.

2. **Technological Factors:** The integration of technology in education has been a significant outcome of globalization. Smart classrooms, e-learning platforms, and digital resources have transformed the teaching and learning experience. Technology has made education more interactive, engaging, and accessible to students nationwide. Therefore, technology forced us to make modifications in the present curriculum. For example, the extensive accessibility of digital learning tools and online resources has led to a greater focus on blended learning and personalized instruction.
3. **Globalization:** In the context of education, globalization refers to the interconnectedness and exchange of ideas, knowledge, and resources across borders. It has facilitated the integration of education systems worldwide, enabling collaboration, innovation, and sharing of best practices. Globalization has opened up new avenues for educational growth and development in India. The increasing interconnectedness of the world has led to a greater emphasis on global education and cultural competency in school curriculums.
4. **Economic Factors:** Economic factors such as changes in job markets, workforce needs, and funding can also impact curriculum changes. For example, a shift towards more service-oriented jobs may prompt a greater emphasis on vocational training in schools.
5. **Research and Development Factor:** Advances in educational research and development can also influence curriculum changes. For example, new research on how students learn and retain information may lead to changes in teaching methods and curriculum design.
6. **Political Factors:** Political factors such as changes in education policy and government funding can also impact curriculum changes. For example, changes in government leadership or policies may result in changes to education funding or mandates for certain subject areas.

From the foregoing discussion, we can conclude that some aspects of doing mathematics have changed in the last decade, in large part because of technology. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in mathematics itself. Technology makes it imperative that: (1) appropriate calculators should be available to all students at all times; (2) a computer should be available in every classroom for demonstration purposes; (3) every student should have access to a computer for individual and group work; and (4) all students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve

problems. Technology is changing the workplace, the home, and daily life. Moreover, the mathematics a person needs to know has shifted, and new mathematics is being created as technological applications emerge. Yet the teaching of mathematics has remained relatively unchanged. As it has for centuries, mathematics often relies on rote memorization. The objectives of mathematics education must be transformed to meet the critical needs of our society: an informed electorate, mathematically literate workers, an opportunity for all students, and problem- solving skills that serve lifelong learning. Both the content that is being taught and the way it is taught need to be reconsidered and, in many cases, transformed. Educational change involves changing teachers' beliefs and understanding as a prerequisite to improving teaching practices. Research indicates that teachers require a thorough understanding of the meaning of educational change before there is an acceptance and adoption of new programmes and approaches. Curriculum change requires in-school management teams, principals, and SCERT, and NCERT to lead the implementation of change in the school as an organization. Effective curriculum change and implementation requires time, personal interaction, in-service training, and other forms of people-based support (Fullan, 1993). The Department of Education (elementary and secondary) should devise a range of initiatives and programmes of professional development to support the phased implementation of change within schools nationally. These initiatives should be designed to increase the capacity of schools to respond to change and to plan for and implement that change at the individual school level.

In order to realize the changing goals and roles of educational objectives, the curriculum should be conceptualized as a structure that articulates required experiences. For this, it should address some basic questions:

(a) What educational purposes should the schools seek to achieve due to the changing needs?

(b) What educational experiences can be provided that are likely to achieve these purposes?

(c) How can these educational experiences be meaningfully organized?

(d) How do we ensure that these educational purposes are indeed being accomplished?

Further, there is a deep disquiet about several aspects of our educational practice: (a) the school system is characterized by an inflexibility that makes it resistant to change; (b) learning has become an isolated activity, which does not encourage children to link knowledge with their lives in any organic or vital way; (c) schools promote a regime of thought that discourages creative thinking and insights; (d) what is presented and transmitted in the name of learning in schools bypasses vital dimensions of the human capacity to create new knowledge; (e) the "future" of the child has taken centre stage to the near exclusion of the child's "present", which is detrimental to the well-being of the child as well as the society and the nation.

Process of Evaluation of Mathematics Curriculum at School Level

In relation to curriculum, evaluation is the process of making value judgments about the merit or worth of a part or the whole of a curriculum. The nature of a curriculum evaluation often depends on its audience and purpose. The potential audiences include:

- Policymakers and other stakeholders (administrators, teachers, students, parents, communities) to inform future action.
- Donors to attract funding or to report on the utilization of funds.
- Researchers - for international comparison and identification of effective practices.

Evaluation of curricula is typically concerned with the: Impact of the curriculum:

- ✓ on individual students, their needs, their level of engagement and their performance;
- ✓ on society, including the appropriateness of values communicated and attitudes fostered, and the level of public satisfaction;
- ✓ on the economy including labour markets as an indicator of economic development;

- the process through which the curriculum was developed;
- content and design of the curriculum compared with:
  - ✓ recent social, technological, economic, or scientific changes;
  - ✓ recent advances in educational research and educational paradigms; possible future directions for curriculum change.

Approaches of Curriculum Evaluation:

Gatawa (1990) has identified five curriculum evaluation approaches:

3. **Bureaucratic evaluation:** This evaluation is usually initiated by Government or the Ministry of Education. Ministry of Education could evaluate a course of study or subjects taught in school to find out whether they need to be improved or modified.
4. **Autocratic evaluation:** This evaluation focuses on what is considered to be the educational needs of a curriculum government usually ask independent evaluators such as consultants to conduct the evaluation.
5. **Democratic evaluation:** This focuses on the experiences and reactions the curriculum initiators have had with the programs or project being evaluated in this approach the evaluation does not lead to a recommendation to be considered by the program implementers.

6. **Norm-referenced evaluation:** This evaluates students' performance relative to other students' performance the performance of current students core of previous students can be compared.
7. **Criterion-Referenced Evaluation:** Criteria referenced measures students' actual performance and compares it with the objectives of instruction identified with the syllabus.

Self-Check Exercise 3

a. Explain any two factors that affect the change in curriculum.

b. Discuss any two approaches to curriculum evaluation

4.5 Summary

Dear learner, In this unit we have discussed the concept, principles of curriculum construction in mathematics, the selection and organization of content in mathematics curriculum.

4.6 Glossary

Assessment - The process through which the progress and achievements of a learner or learners are measured or judged in compliance with specific quality criteria.

Attained Curriculum - Curriculum which indicates the knowledge, understanding, skills, and attitudes that learners acquire as a result of teaching and learning, assessed through different means and/or demonstrated in practice. It may differ from the intended and the implemented curriculum.

Child-Friendly Environment - Supportive educational and community environment that is inclusive, healthy, friendly, protective, and rights-based. The Child-Friendly School model, developed by UNICEF, promotes inclusiveness, gender sensitivity, tolerance, dignity, and personal empowerment.

Culturally Responsive Curriculum - A curriculum that respects learners' cultures and prior experiences. It acknowledges and values the legitimacy of different cultures, not just the dominant culture of a society, and encourages intercultural understanding. It incorporates cultural aspects into the curriculum, rather than adding them on as an extra or separate module or course.

4.7 Answers to Self-Check Exercises

Self-Check Exercise 1

1. Answer: A curriculum is a tool to accomplish the expected objectives of teaching a specific subject. It can be considered as the sum total of all the experiences gained by a child as a result of various formal as well as informal activities at school, home and in the community. Curriculum is the formal and informal content and process by which learners gain knowledge, develop skills and modify attitudes, appreciate the values.

2. Answer: c) It is related to the professional development of teachers.

3. Answer: c) Principle of teacher-centredness

Self-Check Exercise 2

Answer: There are many principles of curriculum construction but the following are two main principles for curriculum construction.

1. Principle of Child-Centeredness: The child should be a central figure in any scheme of curriculum construction. The curriculum is meant for the students, so it should be child-centered not teacher-centered. Therefore, while constructing the curriculum age, ability, interest, capability, and needs of the learner should be taken into consideration.

2. Principle of Community-Centeredness: After getting an education in the school, a child has to live in society. Therefore, the values, attitudes and systems that are prevailing in the community must be reflected in the curriculum. It should reflect the values of democracy, ethos and main concerns of the country.

Self-Check Exercise 3

1. Answer: The two main factors that affect change in curriculum are:
1. Social Factors: The impact of social factors on education cannot be underestimated. From the influence of family dynamics and socioeconomic status to the role of peer relationships and cultural norms, these factors shape the educational experiences and outcomes of individuals. By recognizing the profound effects of social factors and implementing strategies to address them, we can create a more inclusive and equitable educational landscape that empowers every learner to reach their full potential. The needs and requirements of society are dynamic. The changes in values, beliefs, and expectations can prompt curriculum changes. For example, the increasing importance of technology in our daily lives has led to a greater emphasis on Science, Technology, Engineering, and Mathematics education in school curriculum.
2. Technological Factors: The integration of technology in education has been a significant outcome of globalization. Smart classrooms, e-learning platforms, and digital resources have transformed the teaching and learning experience. Technology has made education more interactive, engaging, and accessible to students nationwide. Therefore, technology forced us to make modifications in the present curriculum. For example, the extensive accessibility of digital learning tools and online resources has led to a greater focus on blended learning and personalized instruction.

2. Answer: Two approaches to curriculum evaluation are as follows:

1. Autocratic evaluation: This evaluation focuses on what is considered to be the educational needs of a curriculum, government usually ask independent evaluators such as consultants to conduct the evaluation.

2. Democratic evaluation: This focuses on the experiences and reactions the curriculum initiators have had with the programs or project being evaluated in this approach the evaluation does not lead to a recommendation to be considered by the program implementers.

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4.9 Terminal Questions

1. Explain the concept and importance of curriculum.
2. Discuss the various principles of curriculum construction.
3. Describe the factors affecting change in mathematics curriculum.
4. Discuss the process of evaluation of mathematics curriculum at school level.
5. Discuss the approaches to curriculum construction.

Unit - 5

Methods of Teaching Mathematics: Project Method, Problem Solving and Laboratory Method

Structure

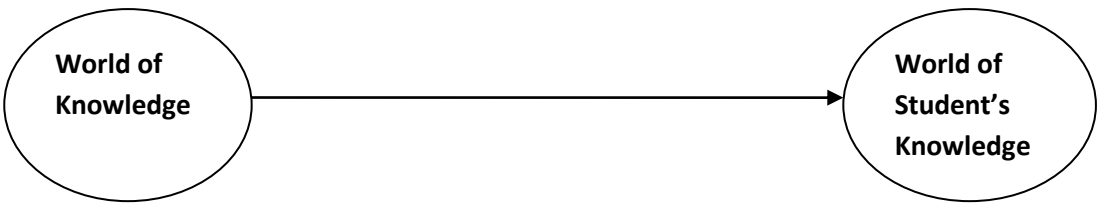
5.1 Introduction

Every Method has some goodness in it, no method is all good. Children should be told as little as possible and induced to discover as much as possible.

-***Valtaire and Spancer.***

Methods of teaching have an intimate relationship with teaching and instructional objectives. So the main aim of teaching is to bring about socially desirable behavioral changes in the children. Though teaching is an art, methods are the way or mode to understand and practice the art. So it is essential that every teacher should be acquainted with different methods of teaching Mathematics.

The word 'Method' has been derived from Latin which means 'Mode' or 'way'. Therefore here it means method of delivering knowledge and transmitting mathematical skills by the teacher to his pupils. Hence the process of interpreting the world of knowledge to pupil's mind is called the method of teaching.



Method of Teaching

The world of knowledge includes; the knowledge, interest, attitude, skill etc. i.e., all the three domains- cognitive, affective and psychomotor.

However it is important to note that a method should not become an end in itself but should be used as a means to achieve the determined aims and objectives of

teaching Mathematics. The teacher is free to use a variety of the teaching methods according to his own abilities, interests and experiences.

5.2 Learning Objectives

After going through this lesson, the learners will be able to:

- Explain the methods of teaching mathematics.
- Understand the Project method.
- Apply the Problem solving method in different situations.
- Understand the utility of laboratory method of teaching mathematics.

5.3 Project Method: Overview, Steps, and Evaluation

The **Project Method** is an instructional approach rooted in **pragmatism** and developed by **Dr. Kilpatrick**. It is grounded in John Dewey's philosophy and emphasizes active, cooperative, and purposeful learning experiences. This method allows students to engage in comprehensive tasks, reflecting real-life activities. It promotes **collaborative work** and involves diverse activities, both within and outside the classroom setting. The students work on a project that requires them to plan, execute, and evaluate their work collectively. This approach aims to provide meaningful learning by engaging students in real-world problem-solving.

Definitions of a Project:

- **Oxford's Advanced Learner's Dictionary**: A project is a plan of action.
- **Ballard**: A project is a bit of real life imported into school.
- **Dr. Kilpatrick**: A project is a unit of wholehearted purposeful activity carried on preferably in its natural setting.
- **Stevenson**: A project is a problematic act carried to completion in its most natural setting.

Basic Principles of Project Method

1. **Psychological Principles of Learning**:
    - Learning by doing
    - Learning by living
    - Association, cooperation, and activity aid in better learning.
2. **Psychological Laws of Learning**:
    - **Law of Readiness**: Learning occurs when a student is prepared to learn.
    - **Law of Exercise**: Repetition of an activity leads to better learning.
    - **Law of Effect**: Actions followed by satisfaction are likely to be repeated.

Steps in the Project Method

1. **Creating the Situation**:
    - The teacher creates a **problematic situation** that captures students' interest. The situation should align with the students' abilities and interests.
2. **Proposing and Choosing the Project**:
    - The teacher stimulates discussion to guide the students in **choosing a project**. The proposed project should meet the **real needs** of the students, and its purpose must be clear.
3. **Planning the Project**:
    - Students, under the teacher's guidance, **plan** the project. Careful planning is essential for the successful completion of the project.
4. **Execution of the Project**:
    - The **execution** phase is where the project is carried out. All students should actively contribute, and this phase is usually the longest.
5. **Evaluation of the Project**:
    - Once the project is completed, the teacher and students jointly **evaluate** the project, assessing if the objectives were achieved.
6. **Recording the Project**:
    - The students maintain a complete **record** of the project, including planning details, discussions, assigned duties, and evaluations. This record helps in reflecting on the learning process.

Example: Running of a Hostel Mess

1. **Recording the number of hostellers.**
2. **Calculating expected expenditure** for the mess.
3. **Allocating expenditures** to students.
4. **Preparing a budget** with class assistance.
5. **Noting account of collections** from students.
6. **Incur actual expenditure**.
7. **Preparing a chart** for a balanced diet.
8. **Fixing and notifying meal times.**
9. **Execution of activities**.
10. **Checking weight of hostellers** at intervals.
11. **Recording punctuality** in activities.
12. **Evaluating the entire program** and finalizing records.

Examples of Mathematics Projects

- Running a **school bank**.
- Managing a **stationary store**.
- **Laying out a school garden**.

- **Planning and estimating the construction** of a house.
- **Planning for an annual camp**.
- **Executing mathematics club activities**.
- **Collecting data** regarding population, birth rate, death rate, etc.

Merits of the Project Method

- **Psychologically grounded**: Promotes learning through activity, association, and cooperation.
- **Develops self-confidence**: Encourages self-discipline and independence.
- **Facilitates critical thinking**: Encourages problem-solving and critical thinking skills.
- **Real-world application**: Prepares students for practical, real-world tasks.
- **Motivates students**: Students engage actively in the learning process, leading to self-motivation.
- **Promotes holistic development**: Provides opportunities for individual and collective development.
- **Encourages discovery learning**: Students learn by doing, which fosters discovery and innovation.

Demerits of the Project Method

- **Time-consuming**: It requires more time compared to traditional teaching methods.
- **Lack of systematic knowledge acquisition**: The knowledge gained might not be structured sequentially.
- **Difficulty in covering syllabus**: Completing the entire syllabus can be challenging using this method.
- **High resource demand**: It requires substantial resources (time, materials, etc.), making it less economical.
- **Limited instructional materials**: Textbooks and other resources may not align well with this method.
- **Inadequate drill and practice**: This method may not provide enough repetitive practice in some subjects.
- **Disorganization in teaching**: The method may lead to a lack of systematic organization in teaching, especially in a fixed curriculum.
- **Doesn’t fit regular timetable**: This method may not always fit within the strict boundaries of regular school schedules.

Self- Check Exercise -1

Q. 1 Which of the following best describes the project method of teaching mathematics?

A) A traditional approach focusing on rote memorization of formulas and procedures.
B) An inquiry-based method where students engage in real-world projects to explore mathematical concepts.
C) A teacher-centered approach where the instructor delivers lectures and assigns textbook exercises. \
D) A method that emphasizes competitive problem-solving among students in a classroom setting.

Q.2 How does the project method differ from traditional approaches to teaching mathematics?

5.4 Problem Solving Method

The child is curious by nature. He wants to find out solutions of many problems which sometimes are puzzling even to the adults. Nevertheless, he must be helped to satisfy his curiosity, whenever possible, by solving various problems. He must teach the pupils how to think so that they are able to transfer these techniques to a vast number of varied problematic situations. Life is full of problems and the successful man in life is he, who is fully equipped with adequate knowledge and reasoning power to tackle these problems. The solution of these problems enables him to have a mastery over his environment. Whenever there is some obstruction in the teaching learning situation, we say that there are some problems. It is a difficulty that is clearly present and recognized by the learners. It may be a purely mental difficulty or it may be physical and involve the manipulation of data. The children recognize it as a challenge.

Definitions

"Problem solving is a set of events in which human being was rules to achieve some goals." **Gagne**

"Problem solving may be defined as a process of raising a problem in the minds of students in such a way as to stimulate purposeful reflective thinking in arriving at a rationale solution." **Risk**

"Problem solving involves concept formation and discovery learning." **Ausubel**

Characteristics of A Problem

- The problem should be meaningful, interesting and practical.
- It should be well defined.
- It should have some educational value.
- As much as possible the problem should be related with the daily life of the child.
- It should be challenging so that the powers of thinking and reasoning can be developed.
- It should have correlation with other study subjects also.

- It should be related with the previous knowledge of the child.
- It should be according to the mental and physical level of the child.
- It should develop imagination and critical power.
- It should develop mathematical skills.
- It should develop scientific attitude amongst the children.

Steps in Problem Solving Method

1. Selection and formation of problem.

The nature of problem should be made very clear to the pupils. The pupil should feel the necessity of finding out the solution of the problem which is selected and formulated. The selection of the problem should be done by the teacher and child both.

2. Presentation of the problem.

After selecting and formulating a problem, teacher should present the problem well before the students. The teacher should also make it clear that how this problem can be solved and how the related data and information can be collected to get the solution of the problem.

3. Formulation of Hypothesis.

Formulation of hypothesis means; preparation of a list of possible reasons of the occurrence of the problem. Formulation of hypothesis develops thinking and reasoning powers of the child. It should be kept in mind that formulated hypothesis must be testable.

4. Collection of relevant data and information.

The child should be stimulated to collect data and information in a systematic and scientific manner. The teacher can suggest many points regarding collection of data to the students. He can ask them to refer extra books and literature.

5. Analysis and organization of Data.

On the basis of collected data and information, the formulated hypothesis are tested. Various statistical techniques are used to analyze and organize the data.

6. Drawing Conclusions.

After analyzing and organizing the data, conclusions are drawn. The selection and rejection of hypothesis are made on the basis of data. Care should be taken that judgements are made only when sufficient data is collected. Discussions and conclusions should be arranged collectively or individually with each child.

7. Testing of conclusions.

No conclusion should be accepted without being properly verified. The student must be asked to be critical while testing conclusions. Thus the correctness of the conclusion is proved by applying them in new or different situations.

Example – Define union of two sets. If A= {2,3,5}, B= {3,5,6} and C = {4,6,8,9}. Prove that A U (B U C)= (A U B) U C

Solution – After selecting and understanding the problem the child will be able to define the problem in his own words that the union of two sets A and B is the set which contains all the members of set a and all the members of set B. the union of two sets A and b is expressed as 'A U B' and symbolically represented as-

A U B = {x:x $\in$ A or x $\Leftarrow$ B}

or x $\in$ (AEB) $\Rightarrow$ x $\in$ A or x $\in$ B.

the common elements are taken only once in the union of two sets.

Step 2- After defining the problem in his own words, the child will analyze the given problem that how the problem can be solved?

Step3- analyzing the various aspects of the problem he will be able to make hypothesis that first of all he should calculate the union of set B and C i.e. B U C. then the union of set A and, B U C. thus he can get the value of A U (B U C). Similarly he can solve (AU B) U C.

Step 4 – then on the basis of given data, the child will be able to solve the problem in the following manner:

In the example it is given that

A = {2,3,5}, B= {3,5,6} and C= {4,6,8,9}

$\therefore$ BUC = {3,5,6} U {4,6,8,9}= {3, 4, 5, 6, 8, 9}

$\therefore$ A U (B U C) = {2, 3, 4} U { 3,4,5,6,8,9} = {2,3,4,5,6,8,9}

Similarly, A U B = {2,3,5} U {3,5,6} ={2,3,5,6}

$\therefore$ (A U B) U C ={2,3,5,6} U {4,6,8,9}={2,3,4,5,6,8,9}

Step5- After solving the problem the child will analyze the result on the basis of given data and verify his hypothesis whether AU (B U C) is equal to (AU B) U C or not.

Step 6- After testing 1 verifying his hypothesis the child will be able to conclude that

AU 9B U C)=(A U B) U C

Thus the child generalizes the result and apply his knowledge in new situations.

Merits of the Problem Solving Method

- This method is psychological and scientific in nature.

- It helps in developing good study habits and reasoning powers.
- It helps to improve and apply knowledge and experiences.
- This method stimulates thinking of the child.
- It helps to develop the power of expression of the child.
- The child learns how to act in new situation.
- It develops group feeling while working together.
- This method helps in minting discipline in the class.
- It develops analytical, critical and generalization abilities of the child.

Demerits of the Problem Solving Method

- This is not suitable for lower classes.
- There is lack of suitable books and reference for children.
- It is not economical.
- It is wastage of time and energy.
- Teachers find it difficult to cover the prescribed syllabus.
- To follow this method talented teachers are required.
- There is always doubt of drawing wrong conclusions.

Self- Check Exercise - 2

Q.1The problem-solving method of teaching mathematics emphasizes the importance of _____ in exploring mathematical concepts.

Q.2In this approach, students are presented with _____ that require critical thinking and creative problem-solving skills.

Q.3Rather than focusing solely on memorizing formulas and procedures, students engage in _____ to solve authentic mathematical problems.

5.5 Laboratory Method in Mathematics Teaching

The **Laboratory Method** is an effective approach used to make mathematics more **interesting** and **meaningful** for students. This method encourages **direct experience** and **active participation**, allowing students to understand mathematical concepts through hands-on experimentation. It provides an environment where students can **verify mathematical facts** and **laws** for themselves using **experiments** and **practical activities**.

Key Features of the Laboratory Method

- **Learning by Doing**: This method follows the maxim of "**learning by doing**," meaning students engage actively in tasks that help them discover mathematical facts through hands-on experiences.
- **Concrete to Abstract**: It begins with **concrete** experiences (using physical materials and equipment) and gradually moves to more **abstract** mathematical concepts.
- **Stimulating Discovery**: The method encourages **self-discovery** by stimulating students' activities and guiding them to explore and verify mathematical concepts on their own.
- **Use of Laboratory Equipment**: The method relies heavily on the use of a well-equipped laboratory, which contains tools such as:
    - **Geometry instruments** (protractors, compasses, etc.)
    - **Mensuration tools** (measuring tapes, rulers, etc.)
    - **Mathematical models** (physical shapes, objects for demonstrations)
    - **Charts and graphs** (for visualizing data)
    - **Balance scales** (for measurement experiments)
    - **Graph paper** and other aids for creating visual representations of data and equations

Procedure of the Laboratory Method

1. **Aim of the Practical Work**:
    - The teacher starts by clearly stating the **objective** or **purpose** of the experiment or activity. This sets a clear direction for the students and informs them about what they are trying to achieve.
2. **Providing Materials and Instruments**:
    - The students are given all the **necessary materials** and **equipment** required to perform the activity. These could include geometric shapes, measuring instruments, graph paper, or other mathematical tools.
3. **Providing Clear Instructions**:
    - The teacher provides **clear instructions** on how to carry out the experiment or task. This step ensures that students understand the procedure and can work independently or in groups to explore the mathematical concept.
4. **Carrying Out the Experiment**:
    - Students then carry out the **experiment** or **activity** as per the given instructions. They use the materials and instruments provided to explore the mathematical concepts in a hands-on manner.
5. **Drawing Conclusions**:
    - After the experiment is completed, the students are required to **draw conclusions** based on their observations. They analyze the results of

the activity and determine how these results verify or illustrate the mathematical concepts they are studying.

Example of Laboratory Method in Mathematics

- **Geometry**: Using **geometric shapes** and tools like **compasses**, **protractors**, and **rulers**, students can explore geometric properties like angles, perimeter, area, etc.
- **Mensuration**: Students may use **measuring tapes** and **scales** to measure real-life objects and then calculate the area, volume, or surface area of different shapes.
- **Graphs**: Using **graph paper**, students can plot data points and create graphs to understand concepts such as functions, slope, and data trends.

Benefits of the Laboratory Method

- **Engagement and Interaction**: The method fosters **active learning**, making mathematical concepts more engaging by allowing students to physically interact with the material.
- **Hands-on Experience**: Students get to experiment and verify mathematical concepts, which reinforces understanding through real-world application.
- **Improves Problem-Solving Skills**: By engaging in experiments, students develop **critical thinking** and **problem-solving** skills, as they must analyze and interpret their findings.
- **Conceptual Understanding**: It allows for a **deeper understanding** of abstract mathematical concepts by connecting them to tangible, real-world objects.

Challenges of the Laboratory Method

- **Resource Intensive**: The method requires well-equipped laboratories, which may not always be available in every school.
- **Time Consuming**: Carrying out experiments and activities can take up more time than traditional lectures.
- **Teacher Expertise**: Teachers need to be well-prepared and knowledgeable to guide students effectively during experiments.

In conclusion, the **Laboratory Method** is a dynamic and effective way to teach mathematics, making abstract concepts more accessible and engaging for students through active participation and hands-on experiments.

Example 1

Derivation of the formula for the volume of a cone.

Aims: to derive the formula for the volume of a cone.

Materials and instruments: cone and cylinders of the same diameter and height, at least 3 sets of varying dimensions, sawdust, water and sand.

Procedure: ask the students to do the following activity.

- Take each pair of cylinder and cone having the same diameter and height
- Note down the diameter and height
- Fill the cone with saw dust / water or sand and empty into the cylinder till the cylinder is full.
- Count the number of times the cone is emptied into the cylinder and note it down in a tabular column.
- Repeat the same experiment with the other two sets of cone and cylinder and note down the reading as before.

S.NO.	DIAMETER OF CONE / CYLINDER	HEIGHT OF CONE/ CYLINDER	NO. OF MEASURES OF CONE TO FILL THE CYLINDER
1	3 CM	5 CM	3
2	5 CM	7 CM	3
3	6 CM	10 CM	3

Drawing conclusions:

Each time, irrespective of the variations in diameter and height it takes 3 measures of cone to fill the cylinder.

Volume of cone = 1/3 volume of cylinder

But volume of cylinder = □ r^2 h

Volume of cone =1/3 □ r^2 h

Example 2:

Sum of three angles of a triangle is 180 degree. “How we can prove this in the laboratory.

Aims:

To prove that sum of the three angles of a triangle is equal to two right angles or 180 degree.

Materials and instruments:

Card board sheet, pencil, scale, triangle and other necessary equipments.

Procedure:

In the laboratory pupils will be given on cardboard sheet each and then they are told how to draw triangles of different sizes on it. After drawing the triangles they cut this separately with the help of scissors.

Observation:

Student will measure the angles of the triangles drawn and write these in a tabular form

Figure no.	Measure of different angles			Total Angle A +B+C
	Angle A	Angle B	Angle C	
1	90	60	30	180

2	120	30	30	180
3	60	60	60	180

Calculation: after measuring the angles of different triangles in the form of cardboard sheet. We calculate and conclude their sum.

In this way by calculating the three angles of a triangle the students will be able to conclude with inductive reasoning that the sum of three angles of a triangle is 180 degree or two right angles.

Some More Topics for Laboratory Method

Derivation of the formula for the

- Circumference of a circle, area of circle
- Area of square, rectangle,, parallelogram, and trapezium
- Area of triangle, right angled triangle, isosceles right angles triangle
- Total surface area of cone, cylinder
- Volume of a sphere
- Volume of a cone
- Expansion of identities such as $(a+b)^2$, $(a-b)^2$, $(a+b+c)^2$
- Verification of Properties of certain geometrical figures like parallelogram, rhombus etc.
- Angle sum property in a triangle
- Congruency postulates
- Theorems relating to triangles, circles and transversal properties.

Merits for Laboratory Method

- The method is based on the principle of learning by doing.
- This method is psychological as we proceed from known to unknown.
- It is based on the student's self-pacing.
- It helps in making clear certain fundamental concepts, ideas etc.
- It develops the self-confidence and teaches the students the dignity of labour.
- The children learn the use of different equipment's, which are used in laboratory.
- It develops in the child a habit of scientific, enquiry and investigation.
- This method presents mathematics as a practical subject.
- It stimulates the interest of the students to work with concrete material.
- It provides opportunities for social interaction and co-operation among the students.
- It is child-centred and therefore it is a psychological method.
- It helps the students to actively participate in the learning process and therefore the learning becomes more meaningful and interesting.

Demerits for Laboratory Method

- This method can be used for a small class only.
- It requires a lot of planning and organization.

- This method is suitable only for certain topics.
- This method it is not possible to make progress quickly.
- This method requires laboratory equipped with different apparatus.
- All mathematics teachers cannot use this method effectively.
- It is an expensive method. All schools are not able to adopt this method.
- This method has very little of theoretical part in it.

Self- Check Exercise - 3

Q.1 What is the primary objective of the laboratory method of teaching mathematics?

A) Memorization of mathematical formulas and procedures.

B) Passive observation of mathematical concepts.

C) Active exploration and experimentation with mathematical principles.

D) Rote learning of mathematical theorems.

Q.2 The laboratory method of teaching mathematics involves students actively engaging in _____ to explore mathematical concepts.

Q.3 In the mathematics laboratory, students have access to various _____ and materials to conduct experiments and investigations.

5.6 Summary

In conclusion we can say that this method is suitable for teaching mathematics to lower classes as at this stage teaching is done with the help of concrete things and examples. Though project method provides a practical approach to learning. It is difficult to follow this method for teaching mathematics. However this method may be tried along with formal classroom teaching without disturbing the school timetable. This method leads to understanding and develops the ability to apply knowledge. The teacher has to work as a careful guide during the execution of the project. It has been suggested that there are many reasons why a problem-solving approach can contribute significantly to the outcomes of a mathematics education. Not only is it a vehicle for developing logical thinking, it can provide students with a context for learning mathematical knowledge, it can enhance transfer of skills to unfamiliar situations and it is an aesthetic form in itself. A problem-solving approach can provide a vehicle for students to construct their own ideas about mathematics and to take responsibility for their own learning. There is little doubt that the mathematics program can be enhanced by the establishment of an environment in which students are exposed to teaching via problem solving, as opposed to more traditional models of teaching about problem solving. The challenge for teachers, at all levels, is to

develop the process of mathematical thinking alongside the knowledge and to seek opportunities to present even routine mathematics tasks in problem-solving contexts.

5.7 Glossary

Project Method: An instructional approach where students engage in hands-on, inquiry-based projects to explore mathematical concepts in real-world contexts.

Laboratory Method: A teaching approach that involves students actively engaging in experiments and activities to explore mathematical concepts.

Problem-Solving Method: An instructional strategy focused on developing students' critical thinking and problem-solving skills through the exploration of mathematical problems.

5.8 Answers to Self- Check Exercise

Self- Check Exercise-1

Answer 1: B) An inquiry-based method where students engage in real-world projects to explore mathematical concepts.

Answer2: Traditional approaches to teaching mathematics often rely on lectures, drills, and textbook exercises, focusing on rote memorization of formulas and procedures. In contrast, the project method encourages active learning through inquiry-based projects where students investigate mathematical concepts in meaningful contexts, fostering deeper understanding and application skills.

Self- Check Exercise-2

Answer 1: Critical Thinking

Answer 2: Problems

Answer 3: Real-world solutions

Self- Check Exercise-3

Answer1: C) Active exploration and experimentation with mathematical principles.

Answer2: experiments and activities

Answer3: tools, equipment, and materials

5.9 References / Suggested Readings

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5.10 Terminal Questions

1) What do you understand by the project method? How can it be utilized for teaching of mathematics?

2) How does the problem-solving method differ from traditional approaches to teaching mathematics?

3) Explain the importance of developing problem-solving skills in mathematics education.

4) Describe a typical mathematics laboratory activity that students might engage in. How does this activity help reinforce mathematical concepts?

5) Explain how the laboratory method promotes hands-on learning and exploration in mathematics education.

6) Discuss the benefits of using manipulative and technology in a mathematics laboratory setting.

Unit - 6

Approaches of Teaching Mathematics: Inductive-deductive, Analytic-synthetic, Heuristic

Structure

6.1 Introduction

Teaching mathematics encompasses a variety of instructional methods and approaches aimed at fostering students' understanding, problem-solving skills, and mathematical reasoning. Different approaches to teaching mathematics recognize the diverse ways in which students learn and engage with mathematical concepts. The traditional approach to teaching mathematics often emphasizes rote memorization of formulas, rules, and procedures. Instruction typically involves teacher-led lectures, textbook exercises, and drills aimed at mastering computational skills. The constructivist approach to teaching mathematics emphasizes active learning and student-centered instruction. Students construct their understanding of mathematical concepts through hands-on activities, exploration, and collaborative problem-solving. Technology integration in mathematics education involves using digital tools, software, and online resources to enhance teaching and learning experiences. Technology facilitates visualization, exploration, and interactive engagement with mathematical concepts. By incorporating diverse teaching approaches, educators can create dynamic and inclusive learning experiences that cater to the needs and interests of all students, fostering deeper understanding and appreciation of mathematics.

6.2 Learning Objectives

After completing this Unit, the Learners will be able to;

- Identify the different approaches of teaching mathematics.
- Understand the Inductive- Deductive approach.
- Apply the Analytic approach in solving different problems.

- Analyze and apply Heuristic approach in different situations.

6.3 Inductive and Deductive Approach

Inductive and deductive is a combination of two separate methods, inductive and deductive. Therefore to understand this method, we have to know about these methods.

Inductive Approach

We still have in our memory the way of proving binomial expression like (a+b) etc. it was of named as method of induction. The way of proving any universal truth by showing that if it is true for any particular case, it is true for the any next case in the same serial order. Inductive method takes in to account the process of induction. In inductive reasoning one proceeds from particular to general, from concrete facts to abstract rules and from specific examples to the general formula. The results are always generalized by studying particular concrete cases and examples.

- E.g. –A small child meets the accidental death of his father. Sometimes later he comes across the death of playing mate and a year later his neighbor uncle passed away. These are special & particular cases of death but they may lead the child to conclude that **"man is Mortal".**
- A child eats a green apple and feets its sour taste, again on an another day he takes another green apple and experiences the same sour taste. The few experiences are enough to make him conclude that **" Green Apples Are Sour".** And hence afterwards whenever he offered green apple he will at once refuse to eat it.

This method of acquiring knowledge is known as " Inductive method". It is nothing but learning from direct experiences.

Teaching of Mathematics with the Help of Inductive Method

1. Knowledge of sum of three angles of a triangle:- Children may be asked to construct a few triangles of any size or shape. Then they may asked to measure and sum up the angles in each case. They may find that the sum is same in each case. Then it may lead to conclude that; "the sum of three angles of any triangle is equal to two right angles."
2. Establishing the formula $(a+b)^2 = a^2 + b^2 + 2ab$:-

The students may be asked to find out the square value in each cases like $(x+y)^2$, $(m+n)^2$,$(p+q)^2$ etc. by simple method of multiplication. After doing these different multiplication, they may be helped in generalizing that

"(1^{st} term + 2^{nd} term)^2= (1^{st} term)^2 +(2^{nd} term)^2+2(1^{st} term)(2^{nd} term)"

i.e. $(a+b)^2 = a^2 + b^2 + 2ab$

3. Establishing the formula of Simple Interest i.e. S.I.= PXRXT/100

The pupils may be asked to solve a few problems on Simple Interest with the help of Unitary method. The summary then may be written on the black board in the following way

S. I. of Rs. 300 for 2 Years at 4% annum = 300X2X4/100= 24.00

S. I. of Rs. 400 for 3 Years at 5% annum = 400X3X5/100= 60.00

S. I. of Rs. 600 for 4 Years at 3% annum = 600X4X3/100= 72.00

A close analysis of above results may lead to conclude that

S.I. = Principal Value X Rate XTime/ 100

i.e. S.I.= PXRXT/100

Merits of Inductive Method

- It is a scientific method because knowledge attained by this method is based on real facts.
- The child gets the knowledge of the purpose of deciding and generalizing laws, formulas etc.
- The knowledge gained by the use of this method is more durable because in this method child himself attains the knowledge by examples, observation and testing.
- The critical observation and logical power of children are developed by inductive method.
- This is a psychological method because many important principles of psychology are used in this method.
- This method guides ths child to the work himself. So this develop self reliance and self confidence in them.
- This method helps to ascertain or establish many laws, relations, formula and new principles of mathematics.
- This method is very useful and suitable for lower classes.
- This method develops curiosity and interest in the child to learn mathematics.
- In this method children themselves attain the knowledge with the help of examples so they don't feel bored (fatigue). They remain active to attain new knowledge.

Demerits of Inductive Method

- This is very slow process, so gaining knowledge by this method costs more time and labour.
- It needs sharp mind, proper planning and enough labour. So it is not easy to attain knowledge by this method for students of all levels.

- This method is useful only for lower classes because syllabus is very wide in higher classes and it is not possible to cover the whole syllabus.
- Only an experienced and able teacher can use this method successfully.
- The ability and capacity of problem solving cannot be developed by the use of this method.
- It is neither easy for teachers nor for students to select or present real examples for generalization.
- Results drawn by the use of this method are not always true. Their truthfulness depends upon a number of examples on which they are based. Because the truthfulness of reliability of any result is more if it is drawn from more number of specific examples.

Deductive Approach

In deductive method one follows deductive reasoning which is just opposite to inductive reasoning as may be seen through the following examples;

1. When the child meets the case of death, he may be told an enquiry that one day or the other, one has to depart. He may have verification of this establishing fact after coming across some other case of death in his life.
2. The child may be told that he should never eat the green apples because they are sour. Afterwards he may verify this fact by tasting some green apples.

In this way deductive reasoning begins with the deductive results or generalized conclusions. Therefore in deductive method one proceeds from general to particular, from abstract rules to concrete cases and from general formula to specific examples.

Teaching Mathematics with the help of Deductive Method

1. The teacher may announce that today he is going to teach S.I. he will then give the relevant formula i. e. S.I.= PXRXT/100

For acquainting the students with its applications, he may also solve a few problems. Then he may ask the students to solve similar problems directly with the help of given formula.

2. The teacher may tell the students that the sum of the three angles of a triangle is equal to two right angles. Afterwards, student may be asked to verify this established fact by measuring the angles of different triangles.
3. Students may be told about the formula of the area of rectangle i.e. A= length X breadth. Then they may be asked to apply it in finding the area of different rectangles.

Merits of Deductive Method:

- By using this method mathematics work becomes very easy and comfortable.
- By deductive method cramming power of students increases.

- By using this method the speed of gaining knowledge increases because students directly use the formula for solving the problem.
- This method should be used when there is shortage of time.
- This method is used for teaching theorem and axioms of geometry, tables in arithmetic etc.
- Both the teacher and pupil do not find any difficulty in using this method.
- More knowledge can be attained in less time by the ise of this method.
- Laws, principles and formulas can easily be checked by using this method.
- Children can do the exercise quickly and easily by using this method.
- This method is short as well as practical.

Demerits of Deductive Method

- This method is not in accordance with psychological principles.
- In this method more emphasis is given on cramming than understanding or discovering.
- In this method students work like machines without knowing the purpose of proceeding in that particular way.
- Knowledge gained by this method is unclear and unstable, because it is not gained by their own efforts.
- In this method there is not scope of developing powers like logical thinking and investigation.
- This method is not suitable for lower classes because it is very difficult for the students of lower classes to understand different formula, laws etc.
- By using this method the teaching learning process becomes uninteresting and dull.
- Children don’t get opportunities to gain new knowledge by using this method.

Self- Check Exercise-1

Q.1 What is the primary difference between the inductive and deductive approaches in teaching mathematics?

A) The inductive approach starts with specific examples and generalizes to a conclusion, while the deductive approach begins with a general principle and applies it to specific cases.

B) The deductive approach involves hands-on activities and experiments, while the inductive approach relies on teacher-led lectures and demonstrations.

C) The inductive approach encourages students to explore real-world problems, while the deductive approach emphasizes abstract reasoning and theoretical concepts.

D) The deductive approach fosters collaborative learning, while the inductive approach promotes independent inquiry and exploration.

Q.2 The _____ approach in teaching mathematics involves deriving general principles or rules from specific instances or examples.

Q.3 In the inductive approach, students analyze patterns, make observations, and formulate _____ based on their findings.

6.4 Analytic and Synthetic Approach

Analytic Approach

First we understand the meanings of “Analysis” and then go through the “Analytic Approach”. According to the Webster Comprehensive Dictionary the “Analysis” means, the resolution of a whole unit into its parts or elements or the process of resolving a problem into its first element (inductive reasoning).

“Analysis is the ability to break down material to its fundamental elements for better understanding of the organization. Analysis may include identifying parts, clarifying relationships among parts and recognizing organizational principles of scientific system”.

Trowbridge

The analytical method proceeds from **unknown to known** facts. In this method the problem is analysed to find out the relations. A statement is analysed into simpler statements and then truth is discovered. It is based on inductive reasoning and critical thinking. All the related facts are analysed to seek help in proceeding to the known conclusion. It is a logical method which leaves no doubt in the minds of students in understanding the core concept and discourages cramming and rote memory of the learner. It facilitates the understanding of the students and motivates them to discover facts by him. It is a psychological method based on the principle of interest, which inculcates the spirit of inquiry and investigation in the students.

Procedure:

If $a/b=c/d$, prove that $(ac-2b^{2)}/b= (c^2-2bd)/d$

The unknown part is $(ac-2b^2)/b= (c^2-2bd)/d$ is true,

if $a\ c\ d – 2\ b^2\ d = b\ c^2 – 2\ b^2\ d$ is true,

if $a\ c\ d = b\ c^2$ is true,

if $a\ d = b\ c$ is true

that is, if $a/b = c/d$ is true,

which is known.

Merits of Analytic Approach

- This method is based on psychological principles.
- The analyse is an explanatory procedure.
- It is based on heuristic approach.
- It develops scientific attitude.
- It leads to the spirit of enquiry and investigation.
- Analysis is the process of thinking.
- This develops self-confidence and logical abilities in the child.
- Knowledge gained by this method is more solid and durable.
- It is a formative method and based on inductive reasoning.

Demerits of Analytic Approach

- This is a lengthy method.
- It is not possible to acquire speed and efficiency.
- Every teacher cannot use this method successfully.
- The whole syllabus cannot be completed within the certain period.
- The use of analytic method is possible only when we have the knowledge of known facts and unknown conclusions.

Synthetic Approach

The "Synthesis" means the assembling of separate or subordinate parts into a new form. It is a process of reasoning from whole to a part and from general to the particular (deductive reasoning). **Webster Comprehensive Dictionary**

The synthesis requires the formulation of new understanding of scientific systems. If analysis stresses the parts, synthesis stresses the whole components of scientific systems may be recognized into new patterns. Unlike analysis, synthesis asks your students to put parts together, to make patterns that one, new to them.

Trowbridge

"It is the process of putting together known bits of information to reach the point where unknown formation because obvious and true."

Katozai

Synthetic approach is just apposite to the analytical method. In this method we proceed from **known to unknown** as synthesis means combing together various parts. In mathematics various facts are collected and combined to find out the result which is unknown. According to Rehman and Katozai there are certain merits and

demerits of the synthetic method. It is a short method and save time in teaching learning process. It is suitable both for intelligent and weak students. But at the other hand, it encourages the memory work and does not develop any reasoning power and students are unable to discover new idea.

Procedure:

The known part is a/b=c/d

Subtract 2b/c on both sides (But why and how the child should remember to subtract 2b/c and not any other quantity)

$a/b - 2b/c = c/d - 2b/c$

or, $(ac - 2\ b^2)/b\ c = (c^2 - 2\ b\ d\ ) / c\ d$

or, $(ac - 2\ b^2)/b\ = (c^2 - 2\ b\ d\ ) /\ d$

Which is unknown.

Merits of synthesis method

- It is a short and quick method.
- It glorifies the memory of the child.
- It formulates, records and presents concisely the discovered facts.
- It omits the trials and errors like in analysis
- This is the method of setting out the solution in a concise form.
- It is informative method it takes less time.

Demerits of synthesis method

- In this method, there is no scope of discovery.
- It leads to rate memory.
- It creates many doubts in the mind of the child.
- There is no opportunity for developing thinking, reasoning, and other mental abilities.
- The recall of each step cannot be possible for every child.

Thus, we conclude that both the methods go together. Analysis help in understanding and synthetic helps in retaining knowledge. The teacher should realise that he may offer help for the analytic form of the solution and that the synthetic work should be left to the pupils (Marwaha, 2009).

Self- Check Exercise-2

Q.1 The analytic approach in mathematics involves solving problems by breaking them down into smaller components and using _____.

A) deductive reasoning

B) synthetic methods

C) step-by-step procedures

D) experimental techniques

Q.2 In the analytic approach, mathematical concepts are analyzed through the use of _____.

A) general principles

B) deductive reasoning

C) specific instances

D) abstract concepts

Q.3 The analytic approach in mathematics emphasizes the use of _____ to solve problems methodically.

Q.4 The analytic approach often involves the application of _____ techniques to derive solutions to mathematical problems.

Q.5 The synthetic approach in mathematics focuses on constructing mathematical concepts through the use of _____.

Q.6 The synthetic approach is commonly used in geometry to prove _____.

Q.7 In the synthetic approach, mathematical concepts are built up from basic _____.

Q.8 The synthetic approach often involves visual representations and _____ to demonstrate geometric relationships.

6.5 Heuristic Method

The word ‘Heuristic’ has been derived from the Greek word ‘Heurisco’ which means ‘I find’ or ‘I discover’. This method implies that the attitude of students shall be that of the discoveries and not of passive recipients of knowledge. ***Armstrong*** originally introduced this method for learning of science. This method emphasis experimentation as the teacher becomes on looker and the child tries to move ahead independently without any help. This method makes the student self-reliant and independent. But the teacher should develop the heuristic attitude by making a lot of preparation. The question should be so planned that it may be possible for the students to find the solution independently by proceeding in the proper direction.

Definition

"This is the method of teaching which places the pupils as far as possible in the attitude of a discoverer." **H. E. Armstrong,**

"The heuristic method is intended to provide training in method. Knowledge is a secondary consideration altogether. **Westaway**

Example 1:

The population of a city is 50,000. The rate of growth in population is 4% p.a. what will be the population after 2 years?

Teacher : what we have to find out in the given question
Student : population after two years.
Teacher : how can we find it?
Student : first we find the population after 1 year.
Teacher : what is the growth of every year?
Student : rate of growth is 4% p.a.
Teacher : what will be the population in the end of first year?
Student : population after 1 year = 50000 + 50000x 4/100
= 50000 + 2000 = 52000
Teacher : what will be the base population for second year?
Student : the base population of second year is 52000.
Teacher : how can we find the growth?
Student : the growth of second year = 52000 x 4 /100 = 2080
Teacher : what will population after two years?
Student : population after two years = 52000 + 2080 = 54080

Merits of Heuristic Method

- This is a psychological method as the student learns by self-practice.
- It creates clear understanding
- It is a meaningful learning
- The student learns by doing so there is a little scope of forgetting
- It develops self-confidence, self-discipline in the students
- The students acquire command of the subject. He has clear understand and notions of the subject.
- It gives the student a sense of confidence and achievement.
- The methods make them exact and bring them closer to truth.
- It inculcates in the student the interest for the subject and also develops willingness in them.

Demerits of Heuristic Method

- It is not suitable for lower classes as they are not independent thinkers. Discovery of a thing needs hard work, patience, concentration, reasoning and thinking powers and creative abilities.
- It is very slow method. That is time consuming method.

- It is lengthy.
- The students have to spend a lot of time to find out minor results.
- The teacher may find it difficult to finish the syllabus in time.
- It does not suit larger classes.
- It suits only hard working and original thinking teachers.
- A method is successful if well-equipped libraries, laboratories and good textbook written in heuristic lines but such facilities are lacking in our school.

Self- Check Exercise-3

Q.1 What is the primary goal of the heuristic method in mathematics?

A) Memorization of formulas and procedures

B) Rote learning of mathematical concepts

C) Encouraging independent problem-solving and discovery

D) Passive observation of mathematical principles

Q.2 In the heuristic method, students are encouraged to:

A) Follow predetermined steps provided by the teacher

B) Rely solely on teacher explanations for problem-solving

C) Explore multiple approaches and strategies

D) Avoid experimentation and exploration

Q.3 The heuristic method in mathematics emphasizes the importance of _____ in problem-solving.

Q.4 In the heuristic approach, students often engage in _____ to explore mathematical concepts independently.

Q.5 Teachers play the role of _____ in the heuristic method, guiding and supporting students' problem-solving efforts.

Q.6 The heuristic method encourages students to ask _____ and seek alternative solutions to mathematical problems.

6.6 Summary

In mathematics education, various teaching methods aim to cultivate students' problem-solving skills and deepen their understanding of mathematical concepts.

The inductive method involves deriving general principles or rules from specific instances or examples, fostering exploration and discovery. Conversely, the deductive method starts with general principles and applies them to specific cases to derive conclusions, emphasizing logical reasoning and deduction. Analytic approaches involve breaking down problems into smaller components and applying step-by-step procedures or algebraic techniques to find solutions. In contrast, synthetic methods focus on constructing mathematical concepts through visual representations, geometric constructions, and deductive reasoning, particularly prevalent in geometry. Meanwhile, the heuristic method encourages independent problem-solving and exploration, emphasizing creativity, critical thinking, and multiple approaches to finding solutions. These diverse methods cater to different learning styles and contribute to a holistic understanding of mathematics, empowering students to become proficient problem-solvers and analytical thinkers.

6.7 Glossary

Inductive Method: An approach to teaching and problem-solving in mathematics where general principles or rules are derived from specific instances or examples.

Deductive Method: An instructional approach in mathematics where general principles or theories are applied to specific cases to derive conclusions or solutions.

Analytic Method: A problem-solving approach in mathematics that involves breaking down problems into smaller components and applying step-by-step procedures or algebraic techniques to find solutions.

Synthetic Method: A method of teaching mathematics that focuses on constructing mathematical concepts through visual representations, geometric constructions, and deductive reasoning.

Heuristic Method: An instructional approach that encourages independent problem-solving and exploration, emphasizing creativity, critical thinking, and multiple approaches to finding solutions.

6.8 Answers To Self- Check Exercise

Self- Check Exercise-1

Answer1: A) The inductive approach starts with specific examples and generalizes to a conclusion, while the deductive approach begins with a general principle and applies it to specific cases.

Answer2: Inductive

Answer3: Hypotheses

Self- Check Exercise-2

Answer1: C) step-by-step procedures

Answer2: B) deductive reasoning

Answer3: logic and reasoning
Answer4: algebraic or symbolic
Answer5: geometric constructions
Answer6: geometric theorems
Answer7: principles or axioms
Answer8: diagrams or illustration

Self- Check Exercise-3

Answer1: C) Encouraging independent problem-solving and discovery
Answer2: C) Explore multiple approaches and strategies
Answer3: creativity and critical thinking
Answer4: exploration or experimentation
Answer5: facilitators or mentors
Answer6: questions or inquiries

6.9 References / Suggested Readings

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6.10 Terminal Questions

Q.1 Estimate the relative importance of the analytic and synthetic methods for teaching of mathematics?

Q.2 Discuss briefly Inductive-Deductive methods for teaching of mathematics with examples?

Q.3 Discuss heuristic method of teaching of mathematics. Illustrate its application with suitable examples?

Unit - 7

Techniques and strategies of teaching Mathematics: Drill and Practice, Assignment, Homework, Supervised Study, Play Way Technique, and Activity Based Technique

Structure

7.1 Introduction

In the realm of mathematics education, a diverse array of approaches and techniques are employed to facilitate effective teaching and learning experiences. These approaches encompass a spectrum of methodologies tailored to accommodate the varied learning styles and abilities of students. From traditional methods emphasizing rote memorization to modern strategies promoting inquiry-based learning, each approach serves a unique purpose in fostering mathematical understanding and proficiency. Techniques such as problem-solving, inquiry-based exploration, hands-on experimentation, and technology integration play pivotal roles in engaging students, encouraging critical thinking, and deepening conceptual comprehension. Moreover, the adoption of heuristic methods, which prioritize independent exploration and creativity, alongside deductive and inductive reasoning, provides students with invaluable opportunities to develop problem-solving skills and cultivate a deeper appreciation for the inherent beauty and logic of mathematics. By employing a multifaceted approach that embraces innovation, collaboration, and adaptability, educators empower students to navigate the complexities of mathematics with confidence, curiosity, and enthusiasm.

7.2 Learning Objectives

After completing this Unit, the learners will be able to;

- Know the various techniques and strategies of teaching mathematics.
- Understand Drill and practice in mathematics.
- Analyze the importance of Assignment, Homework and supervised study.
- Importance of activity based learning.

7.3 Drill and Practice Method in Mathematics Teaching

Drill and Practice are crucial components in the process of learning mathematics, particularly for mastering basic skills and facts. This approach emphasizes repetitive practice to reinforce mathematical concepts and processes until they become automatic. It is grounded in the psychological principles of **learning by doing** and the **law of exercise**, which states that the more a learner practices, the stronger the connection to the learned skill becomes.

The Role of Drill in Mathematics Learning

Drill plays an essential role in learning because it:

- **Facilitates Memorization**: Helps students quickly memorize mathematical facts, formulas, and procedures.
- **Automates Processes**: Repeated practice leads to the **automaticity** of skills, which is essential for more complex problem-solving.
- **Supports Self-Learning**: Encourages students to independently improve their speed and accuracy in solving problems.

It's important to remember that while drill work is essential for developing fluency, it must be preceded by a solid understanding of the concepts. Otherwise, drill work may become ineffective and turn into mere **rote memorization**.

How to Make Drill Effective

1. **Motivated and Engaging**:
    - Drill exercises should be **well-motivated**, making them engaging and relevant to students' learning goals.
2. **Differentiated for Abilities**:
    - Drill activities should cater to **different ability levels** within the class, allowing all students to work at their own pace.
3. **Brief and Distributed**:
    - Drill sessions should be **brief** and spread over time rather than crammed into a single long session, ensuring that students can focus and retain the material better.
4. **Variety in Problems**:

    - **Diverse problems** will keep students engaged and prevent the drill from becoming monotonous. A variety of strategies can help reinforce the concept from different angles.
5. **Progressive Difficulty**:
    - The drill should increase in difficulty as students master earlier skills, which ensures that they are always challenged.
6. **Sufficient Material**:
    - There should be enough material to keep all students **actively engaged** throughout the drill period, preventing boredom or disengagement.
7. **Specific Focus**:
    - Drill exercises should be **targeted** at specific skills or concepts. Focusing on particular details ensures that students get the practice they need for mastery.
8. **Immediate Correction of Mistakes**:
    - It's essential to **detect mistakes early** and correct them to prevent students from practicing errors.
9. **Enjoyment**:
    - The drill work should be **fun** and engaging, making students feel positive about practicing and mastering the skills.
10. **Summarization and Reflection**:

- After completing the drill, the teacher should encourage students to reflect on the practice, summarizing what they have learned and identifying areas for improvement.

Importance / Advantages of Drill Work

1. **Long-Term Retention**:
    - Repeated practice helps **retain material** for a longer time, making it easier for students to recall information when needed.
2. **Ideal for Beginners**:
    - Drill is particularly effective for **beginners**, as it allows them to build a strong foundation of basic skills.
3. **Adjustable Speed**:
    - The **speed** of learning can be adjusted according to each student's ability, ensuring that students don't feel overwhelmed or bored.
4. **Improved Accuracy**:
    - Through repeated practice, students' **accuracy** improves, which is especially critical in mathematical calculations.
5. **Memory Check**:
    - Drill exercises can help teachers **check** how well a student remembers and applies the material.
6. **Correcting Pronunciation**:

    - In some mathematical contexts, such as in the case of learning **mathematical terminology** or concepts, drills can help correct pronunciation or verbalization.
7. **Economical and Efficient**:
    - Drill is a **cost-effective** and time-efficient technique, as it does not require extensive resources, and results can be quickly observed.
8. **Immediate Reinforcement**:
    - Drill provides **immediate reinforcement** of concepts, allowing students to immediately apply what they have learned.

Disadvantages of Drill Work

1. **Not Suitable for All Topics**:
    - Drill work is not effective for teaching **complex concepts** that require deep understanding or creative thinking. It is more suited to basic facts and operations.
2. **Disruptions in Other Classes**:
    - In some cases, drill work can create **disturbances** in other classes, especially when it involves loud verbal exercises or group activities.
3. **Requires Clear Voice**:
    - Effective drill work often requires a **clear and loud voice** from the teacher, especially when the class involves verbal drills or reading exercises.
4. **Can Become Repetitive**:
    - Without proper management, drill work can turn into a **boring routine**, where students may not benefit from the practice due to a lack of engagement or understanding.
5. **Risk of Futility**:
    - If the practice isn't meaningful or if it's not followed by reflection, drill work can become an **academic futility** where students memorize without understanding, limiting long-term learning.
6. **Requires Careful Supervision**:
    - Drill work requires **constant supervision** and careful questioning to ensure that students are practicing the right skills and not reinforcing mistakes.

Conclusion

While **Drill and Practice** can be an invaluable method for reinforcing basic mathematical skills, it must be implemented with care. The key is to ensure that the students understand the underlying concepts before engaging in repetitive practice. When done effectively, drill work can **increase speed**, **accuracy**, and **retention**, making it an essential part of a well-rounded mathematics curriculum. However, it

should always be accompanied by meaningful learning and reflection to ensure it leads to genuine understanding.

Self- Check Exercise-1

Multiple Choice Questions (MCQs):

Q.1 What is the primary objective of drill and practice in mathematics education?

A) Fostering creativity and exploration

B) Encouraging independent problem-solving

C) Reinforcing mathematical skills and concepts

D) Promoting collaborative learning experiences

Q.2 Which of the following is a characteristic feature of drill and practice activities?

A) Open-ended exploration

B) Critical thinking challenges

C) Repetition and reinforcement

D) Hands-on experimentation

Q.3 Drill and practice activities involve repetitive _____ of mathematical skills or procedures to enhance mastery.

Q4. These activities are often used to reinforce _____ operations, such as addition, subtraction, multiplication, and division.

Q.5 The primary goal of drill and practice is to promote _____ and automaticity in mathematical computation.

Q.6 Educators may use various resources, such as worksheets, flashcards, or online _____ platforms, for drill and practice activities.

7.4 Assignment Method

An assignment is work assigned to students either before or after a lesson, and it can be completed at school or at home. It is a type of commitment or responsibility that the student takes on to complete the given task. Assignments should be concise

to encourage students to attempt them. In mathematics, assignments typically include two types of problems:

- **Repetitive Problems**: These focus on new material. By giving problems on different topics, teachers provide students with variety, which can make the assignment more engaging. Repetitive problems help reinforce newly learned concepts and give students a chance to assess their mastery of the topic.
- **Review Problems**: These revisit concepts and skills from earlier lessons, also known as spiral assignments. These assignments combine both repetitive and review problems, ensuring that the student doesn't forget previously learned material. Careful selection of these problems is important.

Purposes of Assignments in Mathematics:

- To solve mathematical problems.
- To create illustrations for a specific topic.
- To collect data related to mathematics.
- To explore the background of a mathematical concept.
- To develop problems based on a topic.
- To undertake mathematical projects.
- To apply mathematical knowledge to problem-solving.
- To generate interest in mathematics.
- To enhance problem-solving skills.
- To cultivate a habit of regular practice and exercise.
- To link the student’s past knowledge and experiences with new learning.

Characteristics of a Good Assignment:

- It should help overcome the student's difficulties.
- The assignment should be clear and definite.
- It should stimulate and guide the student's learning activities.
- It should connect with the student's previous knowledge and experiences.
- Individual differences must be taken into account.
- It should be motivating and engaging.
- The assignment should provide appropriate feedback and reinforcement.

How to Make an Assignment Effective:

- An assignment should not just involve dictating questions; it should include hints to help the students succeed.
- Assignments should be insightful and clarify doubts or misunderstandings.
- The assignment should be motivating and collaborative, with active participation from both the teacher and the student.
- Interaction between the teacher and student is essential.

- Textbooks, reference books, and other teaching materials should be used to guide and stimulate the student's assignment activities.
- The assignment should be centered around activities, and be need-based and interest-driven.
- Teachers should be aware of any challenges that may arise.
- Proper planning of assignments is essential.
- The assignment tasks should be focused and specific, as vague or lengthy assignments may not lead to better outcomes.

Self- Check Exercise-2

Q.1 What is the primary purpose of the assignment method in mathematics education?

A) Encouraging collaborative problem-solving

B) Reinforcing mathematical concepts through independent practice

C) Fostering creativity and exploration

D) Promoting hands-on experimentation

Q.2 Which of the following is a characteristic feature of the assignment method?

A) Group-based activities

B) Open-ended exploration

C) Teacher-centered instruction

D) Individualized tasks

Q.3 The assignment method involves giving students _____ tasks to complete independently.

Q.4 Teachers may assign homework, worksheets, or practice problems as part of the _____ method.

Q.5 The primary goal of the assignment method is to provide students with opportunities for _____ practice and reinforcement of mathematical skills.

Q.6 Assignments are often tailored to students' _____ levels and learning needs.

7.5 Homework

In modern days the curriculum in secondary schools is so vast, that school time is not sufficient to cover everything provided in the curriculum. So if the teachers want to do justice with the curriculum they have to counterpart with homework. It has to be given regularly. Homework plays a vital role, as the teachers get very short time to cover the heavy load of the curriculum so under the circumstances; it is not only important but also essential to give homework to the students. The homework in mathematics may consist of some problems based on facts taught in the classroom. Students may be asked to learn certain principals, definitions, facts, draw graphs, charts, tables etc. by giving homework means creating in the children a study environment at home. The nature and amount of homework should be given according to the capacities of the children. Homework shoul assist as a part of internal assessment and proper weightage should be given.

Objectives of Home Work

- It utilizes the leisure time of the child; otherwise the child will waste it in gossiping.
- It cultivates the habit of regularity and hard work among the children.
- It provides the opportunity of independent work.
- It provides opportunity for the application and the practice of the gained knowledge.
- It supplements the classroom teaching.
- It acts as link between parents and teachers.
- It creates an environment of school feeling at home among the children.

Principles of Homework:

- Principle of accuracy.
- Principle of interest.
- Principle of clearness.
- Principle of relevance.
- Principle of economy.
- Principle of sequence.

Importance/ Advantages of Homework in Mathematics:

- It brings about closure relationship among the parents and the school
- It promotes the habit of self-study in children.
- It develops a sense of responsibility among the children.
- The constant anxiety of doing homework promotes the progress of the child.
- It provides opportunity to utilize the pleasure time of the students.
- It develops self-confidence and self-reliance amongst the students.

Disadvantages of Homework in Mathematics:

- It takes too much time of the children after the school.
- Homework does not provide suitable conditions for work.

- Many pupils involve their parents or others to complete their homework.
- Some children develop emotional tensions because of homework.
- Homework assignments are sometimes misused as punishments.
- It also deprives the children of their leisure time.
- Homework may adversely affect the health of the children.
- Load of homework in one subject may affect the achievement in other subject.

Precautions while assigning Homework in Mathematics:

- Do not assign the problems to the children discriminately.
- Do not assign too many problems at a time.
- Homework should be given in brief, so that the pupils will be more willing to try to complete it.
- Only the problems on the newly developed topic of the day should not be given in homework.
- Homework should not be used as a punitive device.
- Clear direction should be given to the students.

Self- Check Exercise-3

Q.1 What is the primary purpose of the homework method in mathematics education?

A) Reinforcing mathematical concepts through independent practice outside the classroom

B) Fostering collaboration among students

C) Providing opportunities for hands-on experimentation

D) Encouraging creativity and exploration

Q.2 Which of the following is a characteristic feature of the homework method?

A) Group-based activities

B) Teacher-centered instruction

C) Independent practice

D) Hands-on exploration

Q.3 Homework assignments are tasks given to students to complete _____ the classroom.

Q.4 The primary goal of the homework method is to provide students with additional _____ for practicing and reinforcing mathematical skills.

Q.5 Homework assignments typically involve tasks such as solving problems, completing exercises, or reviewing _____.

Q.6 Homework assignments should be appropriately _____ to students' learning needs and abilities.

7.6 Supervised Study

Supervised study in mathematics refers to a structured learning environment where students work independently on mathematical tasks or assignments under the guidance and supervision of a teacher or instructor. During supervised study sessions, students have the opportunity to consolidate their understanding of mathematical concepts, practice problem-solving skills, and receive personalized assistance and feedback from the teacher. The teacher may provide guidance, clarification, and support as students work through challenging problems, review class materials, or complete homework assignments. Supervised study sessions may take place during designated class periods, after-school sessions, or in dedicated study halls. This approach allows students to work at their own pace, seek assistance when needed, and develop confidence in their mathematical abilities with the support of their teacher. Supervised study in mathematics aims to enhance students' mathematical proficiency, foster independent learning skills, and promote a deeper understanding of mathematical concepts through guided practice and reinforcement.

It is useful for the teaching of understanding level. In this technique, both the teacher and the child remain active. In this, every child has to devote his prescribed time for self-study. It creates a formal atmosphere for the self-study. The child learns according to his abilities and capacities. The teacher supervises the activities of the child.

Forms of Supervised Study:

It may be in the form of conference plan, special teacher plan, divided period plan, double period plan and periodic plan.

Steps for supervised study:

- Preparation for the study.
- Instructions for the study.
- Supervision by the teachers.
- Development of blackboard summary.

Importance/ Characteristics of Supervised Study

Supervised study plays a crucial role in supporting students' academic success and fostering a positive learning environment. Here are some key reasons highlighting its importance:

Guidance and Support: Supervised study provides students with access to guidance and support from qualified teachers or instructors. This ensures that students receive assistance when they encounter difficulties, helping them overcome challenges and progress in their learning journey.

Individualized Attention: In supervised study sessions, teachers can provide individualized attention to students, addressing their unique learning needs and offering personalized feedback. This tailored approach promotes a deeper understanding of concepts and allows for targeted intervention when necessary.

Accountability: Supervised study instils a sense of accountability in students, as they are expected to complete assigned tasks or work on specific academic goals under the supervision of a teacher. This accountability helps students stay focused and motivated to achieve their learning objectives.

Practice and Reinforcement: Supervised study provides students with opportunities for practice and reinforcement of learning. Through guided practice sessions, students can consolidate their understanding of concepts, develop problem-solving skills, and reinforce newly acquired knowledge.

Monitoring Progress: Supervised study allows teachers to monitor students' progress closely, track their academic performance, and identify areas for improvement. This on-going assessment enables teachers to provide timely support and intervention to help students succeed.

Promoting Independence: While supervised study offers support and guidance, it also encourages students to develop independence in their learning. By working on tasks independently under the supervision of a teacher, students learn to take ownership of their learning and develop essential self-regulation skills.

Creating a Positive Learning Environment: Supervised study sessions contribute to creating a positive and conducive learning environment where students feel supported, valued, and motivated to excel academically. The presence of a teacher fosters collaboration, engagement, and a sense of community among students.

Overall, supervised study is instrumental in facilitating academic growth, fostering student success, and nurturing a culture of continuous learning and improvement in educational settings.

Precautions while using Supervised Study:

While supervised study can be highly beneficial for students, it's important to take certain precautions to ensure that it remains effective and conducive to learning. Here are some precautions to consider:

Clear Expectations: Clearly communicate expectations for behaviour, participation, and academic integrity during supervised study sessions. Ensure that students understand the purpose of the session and the tasks they are expected to complete.

Teacher Availability: Ensure that a qualified teacher or instructor is present during supervised study sessions to provide guidance, support, and supervision. Teachers should be available to answer questions, clarify concepts, and offer assistance as needed.

Safe Environment: Create a safe and conducive learning environment where students feel comfortable and supported. Ensure that the study area is well-lit, organized, and free from distractions to promote focus and concentration.

Structured Activities: Provide structured activities or assignments for students to work on during supervised study sessions. Clearly outline the tasks to be completed and provide guidance on how to approach them effectively.

Individualized Support: Recognize that students may have different learning needs and abilities. Offer individualized support and assistance to students based on their specific requirements, learning styles, and academic goals.

Regular Monitoring: Monitor students' progress and behaviour during supervised study sessions to ensure that they are staying on task and making effective use of their time. Address any issues or concerns promptly to prevent disruptions and maintain a productive learning environment.

Feedback and Assessment: Provide regular feedback and assessment on students' work to help them track their progress and identify areas for improvement. Encourage students to reflect on their learning and set goals for future study sessions.

Respect for Boundaries: Respect students' privacy and boundaries during supervised study sessions. Avoid pressuring students or invading their personal space, and maintain professionalism at all times.

By taking these precautions, educators can ensure that supervised study sessions are effective, supportive, and beneficial for students' academic growth and success.

More freedom and facility for mutual exchange, teacher should possess insights and resourcefulness etc.

Self- Check Exercise-4

Q.1 What is the primary purpose of supervised study in mathematics education?

A) Fostering creativity and exploration

B) Reinforcing mathematical concepts through independent practice

C) Providing opportunities for hands-on experimentation

D) Promoting collaborative problem-solving

Q.2 Which of the following is a characteristic feature of supervised study?

A) Group-based activities

B) Individualized guidance

C) Open-ended exploration

D) Teacher-centered instruction

Q.3 Supervised study provides students with guidance and _____ from qualified teachers or instructors.

Q.4 During supervised study sessions, students work independently on tasks such as completing homework assignments, reviewing class materials, or practicing _____.

Q.5 The presence of a teacher during supervised study sessions allows for _____ assistance and feedback to students as they work through academic tasks.

Q.6 The primary goal of supervised study is to help students reinforce their understanding of mathematical concepts through _____ practice and review.

7.7 Play-Way Method

The play-way method of teaching mathematics integrates playful activities and games into the learning process to make mathematical concepts more engaging and accessible to students. This approach capitalizes on children's natural inclination to play and explore, leveraging games, puzzles, and hands-on activities to introduce and reinforce mathematical concepts in a fun and interactive manner. Rather than relying solely on traditional instructional methods, the play-way method encourages

active participation, creativity, and problem-solving skills development through playful exploration. Activities may include math games, manipulative-based tasks, outdoor activities, and role-playing scenarios that incorporate mathematical concepts such as counting, sorting, patterns, geometry, and arithmetic operations. By incorporating play into the learning process, the play-way method fosters a positive attitude towards mathematics, enhances conceptual understanding, and promotes a love for learning in young learners. Additionally, it caters to diverse learning styles and abilities, allowing students to explore mathematical concepts at their own pace while experiencing enjoyment and satisfaction in the learning process. Overall, the play-way method of teaching mathematics creates a dynamic and enriching learning environment that nurtures students' mathematical development and fosters lifelong learning skills.

Play-way method of teaching mathematics involves a serious teaching along with playing games. This method was introduced by **Froebel** who is known as **"Father of Kindergarten method".** The Mathematics teacher should provide some simple games and make the child to play for acquiring math. Usually, this method is more suitable for primary level students. Obviously, mathematics is taught from our childhood at the age of four or five. For a better future, a strong foundation should be laid down at this particular age. Mainly, the teacher has to teach arithmetic skill to this age group of students. So, it can be easily incorporated with games. The teacher can use as many apparatus for gaming such as toys and playing objects. This will make the child to learn mathematics with more desire. By learning in play-way method, the child learns mathematics clearly and more easily. Thus learning will not be burden for them.

There is a theory, which states that a child understands his needs and goals while playing. So it is very important to teach children with the play way method. It has been proved that maximum amount of learning results while playing games. While playing games the environment is very relaxed, this makes learning interesting and fun. This is the most desirable method of learning for kids. The informal and free atmosphere gives the kids a chance to learn concepts, ideas, math and even language. Toys can sometimes be the root of mathematical concepts. Thus, in the play way methodology toys and apparatus like checkers, magic squares, puzzles and building blocks are used to make teaching and learning a memorable experience for the learners.

Advantages/ Merits of Play way Method:

- It is a psychological method.
- Helps in the all-round development of the personality.
- Proves an effective activity centred method.
- Helps in gaining useful and permanent knowledge.
- Provides opportunities for healthy competition.
- Helps in reducing fatigue.

- Development of group feelings.

Limitations/ Demerits of Play way Method:

- Possibility of going astray.
- Danger of losing play way spirit.
- Wastage of time and energy.
- Not suitable to the shy and unwilling students.
- Possibility of utilizing unfair means.

Self- Check Exercise-5

Q.1 What is the primary objective of the play-way method in teaching mathematics?

A) Memorization of mathematical formulas

B) Promotion of playful exploration and engagement

C) Rote learning of arithmetic operations

D) Emphasis on traditional instructional methods

Q.2 Which of the following is a characteristic feature of the play-way method?

A) Teacher-centered instruction

B) Emphasis on memorization and repetition

C) Integration of games and hands-on activities

D) Passive learning experiences

Q.3 The play-way method of teaching mathematics encourages _____ exploration and discovery to enhance learning experiences.

Q.4 Activities such as math games, puzzles, and hands-on tasks are commonly used to introduce and reinforce mathematical concepts in the _____ method.

Q.5 The play-way method fosters a positive attitude towards mathematics by making learning enjoyable and _____ for students.

Q.6 By incorporating play into the learning process, the play-way method caters to diverse learning _____ and encourages active participation.

7.8 Activity Based Teaching

Activity Based Teaching (**Also known as Activity Learning Method)** is another method to be used for teaching mathematics to the students of primary level. This method was adopted by an English man David Horsburgh. This method made the students as an active learner and not as a passive learner. Here the teacher should provide activities and experiments for each subject theme. The activity should be appropriate to several constraints like age of the students, concept of the subject. The activity should not deviate the child from learning the concepts. Such activities give a pleasure of learning and develop self-reliance in students. Gradually, it encourages the student to learn new concepts and makes the learning in a joyful manner. Mathematics can be taught using this method in a niche manner. The concept of arithmetic and algebra shall be easily implemented with this method of teaching.

Activity-based teaching is an instructional approach that emphasizes hands-on, experiential learning through engaging activities and tasks. Rather than relying solely on lectures and textbooks, activity-based teaching encourages students to actively participate in their learning by exploring, experimenting, and problem-solving in a practical context. This approach integrates a variety of activities, such as experiments, simulations, games, projects, and group work, to help students develop a deeper understanding of concepts and enhance their critical thinking skills. Activity-based teaching promotes student engagement, motivation, and retention of information by providing meaningful and relevant learning experiences. It also caters to diverse learning styles and encourages collaboration, communication, and creativity in the classroom. Overall, activity-based teaching fosters a dynamic and interactive learning environment that empowers students to take ownership of their learning and become lifelong learners.

Importance:

Engagement: Activity-based learning actively engages students in the learning process, making education more enjoyable and meaningful.

Retention: Hands-on activities promote better retention of information as students experience concepts first-hand rather than just reading or hearing about them.

Understanding: Through activities, students gain a deeper understanding of concepts as they apply theoretical knowledge to real-life situations or practical tasks.

Critical Thinking: Activity-based learning encourages students to think critically, solve problems, and make decisions, fostering the development of essential skills for academic and real-world success.

Collaboration: Many activities involve collaboration and teamwork, promoting communication skills, empathy, and cooperation among students.

Creativity: Activities often allow for creativity and innovation, enabling students to express themselves and explore new ideas in a supportive environment.

Personalization: Activity-based learning can be tailored to students' interests, abilities, and learning styles, providing opportunities for personalized learning experiences.

Characteristics:

Hands-On Experience: Activities provide students with hands-on experience, allowing them to interact with materials, manipulate objects, and perform tasks to deepen their understanding.

Active Participation: Students actively participate in the learning process through exploration, experimentation, and problem-solving, rather than passively receiving information.

Varied Formats: Activity-based learning encompasses a variety of formats, including experiments, simulations, games, projects, discussions, and role-plays, catering to diverse learning preferences.

Real-World Application: Activities often involve real-world applications of concepts, helping students see the relevance and practical implications of what they are learning.

Feedback: Activities provide opportunities for immediate feedback, allowing students to assess their understanding and progress, and teachers to identify areas for further instruction or support.

Inclusivity: Activity-based learning can be inclusive, accommodating students with different learning needs, abilities, and backgrounds by providing multiple entry points and pathways to success.

Student-Centered: Activity-based learning is student-centered, focusing on the needs, interests, and strengths of individual learners, and empowering them to take ownership of their learning journey.

By incorporating activity-based learning into the curriculum, educators can create dynamic and engaging learning environments that inspire curiosity, foster creativity, and cultivate lifelong learners.

Comparison between Play-Way Method and Activity Bases Method

Play-way Method	**Activity Based Teaching**
High range of fun	Somewhat less when compared to play-way method
Engages indirect learning	Engages direct learning
Indirect involvement in subject	Direct involvement in subject
Gives more fun to the students	Less fun to the students
Active participation of Students	Active participation of Students
Games are provided	Activities are provided

Self- Check Exercise - 6

Q.1 What is a key benefit of activity-based learning?

A) Passive engagement

B) Memorization only

C) Hands-on experience

D) Strict teacher control

Q.2 Which of the following is a characteristic of activity-based learning?

A) Teacher-centered approach

B) Limited student participation

C) Real-world applications

D) Rote memorization

Q.3 Activity-based learning encourages _____ participation and engagement in the learning process.

Q.4 Hands-on _____ allows students to interact with materials and concepts in a tangible way.

Q.5 Real-world _____ of concepts help students understand their practical applications.

Q.6 Activity-based learning fosters _____ thinking and problem-solving skills.

7.9 Summary

These methods of teaching have their own merits and demerits. There is no compulsion in adopting any method. But the teacher should decide the suitable method that makes the process of learning and teaching in a reasonable way. The right method used to teach delivers the depth of the subject to the students and does not vanish the interest of the students to learn mathematics subject.Many teachers use inappropriate methods to teach the students without concerning about the learner's mind and circumstances. Surely, it will reflect later but in a serious manner. This situation should be removed and proper measures have to be taken for the welfare of the students.

7.10 Glossary

Drill: A method of teaching that involves repetitive practice of skills or procedures to reinforce learning, typically through exercises or tasks focused on specific concepts or techniques.

Assignment: A task or activity given to students to complete, often as part of their coursework or as additional practice to reinforce learning outside the classroom.

Homework: Assignments or tasks given to students to complete outside of class time, typically to reinforce learning, practice skills, or prepare for upcoming lessons.

Supervised Study: A structured learning environment where students work independently on academic tasks or assignments under the guidance and supervision of a teacher or instructor.

Play-way Method: An instructional approach that integrates playful activities, games, and hands-on experiences into the learning process to make educational content more engaging and accessible to students.

Activity-Based Learning: An instructional approach that emphasizes hands-on, experiential learning through engaging activities, tasks, and projects, encouraging active participation, critical thinking, and problem-solving skills development.

7.11 Answers To Self- Check Exercise

Self- Check Exercise-1

Answer1: C) Reinforcing mathematical skills and concepts

Answer2: C) Repetition and reinforcement

Answer3: practice

Answer4: basic arithmetic

Answer5: fluency

Answer6: software

Self- Check Exercise-2

Answer1: B) Reinforcing mathematical concepts through independent practice

Answer2: D) Individualized tasks

Answer3: individual

Answer4: assignment

Answer5: independent

Answer6: proficiency

Self- Check Exercise-3

Answer1: A) Reinforcing mathematical concepts through independent practice outside the classroom

Answer2: C) Independent practice

Answer3: outside

Answer4: opportunities

Answer5: concepts

Answer6: tailored

Self- Check Exercise-4

Answer1: B) Reinforcing mathematical concepts through independent practice

Answer2: B) Individualized guidance

Answer3: supervision

Answer4: problem-solving

Answer5: personalized

Answer6: independent

Self- Check Exercise-5

Answer1: B) Promotion of playful exploration and engagement

Answer2: C) Integration of games and hands-on activities

Answer3: playful

Answer4: play-way

Answer5: engaging

Answer6: styles

Self- Check Exercise-6

Answer1: C) Hands-on experience

Answer2: C) Real-world applications

Answer3: active

Answer4: experience

Answer5: applications

Answer6: critical

7.12 References / Suggested Readings

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7.13 Terminal Questions

Q.1 What is the purpose of drill in education, and how does it differ from other teaching methods?

Q.2 How do assignments contribute to students' learning and academic development?

Q.3What are the benefits of homework for students' academic progress?

Q.4 What role does a teacher play in supervised study sessions, and why is supervision important?

Q.5 How does the play-way method enhance student engagement and learning in comparison to traditional teaching methods?

Q.6 How can teachers integrate activity-based learning into their curriculum to enhance student learning outcomes?

Unit - 8

Strategies for Teaching of Mathematics to CWSN (Gifted, Slow Learners, Learners with Dyscalculia). Difficulties Faced by the Teachers in Teaching of Mathematics and Suggestive measures to overcome

Structure

8.1 Introduction

In the diverse landscape of the classroom, educators encounter students with a wide spectrum of abilities and learning styles, including gifted individuals who grasp mathematical concepts quickly and with ease, as well as slow learners who may require additional support and reinforcement to master the same material. Recognizing and addressing the unique needs of both these groups is essential for fostering inclusive and effective mathematics education. For gifted learners, strategies often involve providing enriched and challenging opportunities that extend beyond the standard curriculum, such as advanced problem-solving tasks, independent research projects, or participation in math competitions. These strategies aim to nurture their curiosity, creativity, and passion for mathematics, while also fostering critical thinking skills and a deeper understanding of mathematical concepts. On the other hand, for slow learners, strategies focus on differentiation, scaffolding, and personalized support to accommodate their pace of learning and address areas of difficulty. This may involve breaking down complex concepts into smaller, more manageable steps, providing additional practice and reinforcement, using manipulative or visual aids to enhance understanding, and offering individualized instruction or remedial support as needed. By implementing tailored strategies that cater to the specific needs and abilities of both gifted and slow learners, educators can create inclusive learning environments where all students have the opportunity to thrive and succeed in mathematics.

8.2 Learning Objectives

After completing this unit, the learners will be able to;

- Know the needs of gifted children
- Understand the various strategies to be adopted for all kinds of learners in the classroom.

8.3 Strategies for Teaching of Mathematics to Gifted Children

People, who have special gift for at least at the potential for the gifted performance, may go through life unrecognized. Similarly, sometimes gifted children and youths are not discovered by their families and close associates do not give much training and also does not bring them to light. Especially when the student belong to poor families or minority groups, they may be deprived of chances to demonstrate and develop their potential. We good have more outstanding artist and scientists if every talented child had the opportunity and the training necessary to develop his or her talents to the fullest possible extant.

Definition of gifted Children

" The gifted child is one who excels in creative thinking and abstract reasoning has a wide scope of interest and produces work of super quality." **Carrol and Martens**

" The gifted children rate is far above the average in physic, social adjustment, personality traits school achievements, play, information and versatility of interests." **Terman and Orden**

Education of Gifted Children

When a Gifted child works with general children he develops a sense of superiority complex. They see other students with inferiority feeling and slowly their emotional adjustments disturbs. Keeping different qualities of gifted child in mind the following objectives have been determined for their education.

- Determination of higher level curriculum and learning experiences in the area of giftedness
- Developing their critical abilities.
- Providing them sufficient study martial resources
- Proper development of their abilities and skills.
- Providing them the environment for development of creative and constructive powers
- Providing them circumst5ances to fulfill their curiosity.
- To promote their higher level abilities and experiences.

Programmes and Projects for Gifted Children

Acceleration – the gifted can be provided acceleration in following ways ;

1. Giving them admission in schools before the required age.
2. In one academic session only they can allowed to study the syllabus of two or more classes and promoted to the next class.
3. Allow to finish their course in less time than other average students.

Enrichment programmes –

1. Individual enrichment program.
2. Group Enrichment program.
3. Giving them extra work in math's class.
4. Inspire and motivate them to learn new knowledge related to mathematics.
5. To prepare various new projects in mathematics.
6. Teaching by innovative and discovery methods.
7. To develops interesting lesson in mathematics.
8. To develop constructive and creative lessons in mathematics.

Special schools and classes- gifted children have very sharp mind so they are not able to be taught in the normal classroom of mathematics. So most of the educationists have given the provision for the special schools and classes for gifted children through which these children can:

1. Get opportunity to work according to their own capacities.
2. Can do dashing work /challenging works.
3. On the bases of their extra ordinary abilities they can be thought through special teaching techniques.
4. They can get various opportunities to show their leadership.
5. Will not have to face the problem of promotion.

But in the present situations it is difficult as well as expensive to arrange special schools and classes for gifted students. In NPE,1986, for the free and qualitative education to gifted students opened special schools all over India, they are Nvodaya Vidhyalaya to meet the following objectives.

- To provide education free of cost.
- Provide qualitative education.
- Giving experiences to learn in group.
- Arrangement of special education for high achievers.
- Proper organization of co-curricular activities.
- To motivate the child for self-study.
- To develop various abilities completely.
- To develop self-dependence and self-confidence.
- To organize debates, workshops Excursions.
- All round development of the children.

Self- Check Exercise-1

Q.1 What is a common strategy used to challenge gifted students in mathematics?

A) Providing remedial instruction

B) Offering enrichment opportunities

C) Assigning repetitive drills

D) Using a one-size-fits-all approach

Q.2 Which of the following is a characteristic of effective strategies for teaching gifted children?

A) Simplifying concepts to match their level

B) Encouraging passive learning

C) Focusing solely on rote memorization

D) Tailoring instruction to their advanced abilities

Q.3 Enrichment activities such as _____ projects and math competitions are effective strategies for challenging gifted students in mathematics.

Q.4 Differentiated instruction involves _____ teaching methods and materials to meet the diverse needs of gifted learners.

Q.5 Gifted students benefit from opportunities to _____ with like-minded peers, such as in math clubs or advanced classes.

Q.6 Providing _____ problems and advanced topics beyond the standard curriculum helps keep gifted students engaged and motivated in mathematics.

8.4 Strategies for Teaching of Mathematics to Slow Learners

In the present system of education, students are identified as slow learners purely on the basis of their poor performance in the examination, which, in most cases deviates from what is taught. Consequently even talented students are sometimes misconstrued as dullards. So, a slow learner is one whose performance is very dismal in the examination. He is neither mentally retarded nor is on the lower rungs of intelligence scale.

The reasons for some students learning slowly are innumerable. One of the main reasons is the `no detention system' at the primary and upper primary level. Students are promoted to higher classes on the basis of attendance, even if they score low marks. The heterogeneous composition (mental age & physical age) of overcrowded classes in all government run schools and private schools also produces slow learners. So the incapacity of the teacher to pay individual attention to a student over a long period makes a student a slow learner. A slow learner is thus a product of negligence of school at different stages of learning, inspite of his innate capacity to learn.

There are some problems very specific to the individual. Ill health, lack of concentration, less exposure to the subject taught and parental background are some causative factors for slow learning. Talents differ. A Childs capacity to learn different subjects varies from student to student. For instance, learning mathematics is a knack. All students do not do well in mathematics just as they do in other subjects. While other subjects can be learnt at any stage, it is very difficult for students to learn mathematics without the basics. Students show interest in the subjects they like and neglect other subjects if not taken care of. An urban child learns languages like English well while a rural child cannot, however well the teacher tries to explain.

Teaching math to the slow learner is a challenging task for many parents and tutors. Math may be challenging for slow learner, but not impossible. Students who are slow learner are not learning disabled. They are also like normal students. Slow learner students also want to learn math, but lack of learning ability and math education system they are not able to learn faster. Later, those students considered as a slow learner. Some techniques are there which can help slow learner students.

1) Give Time:

Slow learner needs more time to understand any problem or to find out the answer. Give extra time to slow learner students. This will increase their confidence. Do not pressurize students to perform on time beyond their ability. This will only decrease confidence.

2) Teach in a small group:

Slow learner students need extra attention. With a small students group you can effectively respond each student. **Math Online Tutors** can be the best option for slow learner students.

3) Create Fun atmosphere:

Environment is more potent than willpower. Create a fun environment for students. Use new learning techniques, especially for slow learner students. Parents can provide math games and activities for their children.

4) Develop a helpful plan:

Build a helpful environment for students. Encourage students to ask questions and let them feel free to ask for any help. Provide Math to slow learner students.

5) Concept Building:

Most of the slow learner students face difficult to understand the new concepts. Try to relate the new concepts with previous concepts. This will help them to catch the new concepts relatively fast.

6) Real Examples:

One of the best ways to teach math not only slow learner even for normal students is, explain concepts using real life examples.

7) Provide opportunity:

Whenever possible provide opportunities to show them their work. Let the students teach you about math. This will help students to reduce math fear.

8) Review time to time:

Because slow learner needs more time to understand the concepts. Frequent reviewing can help them out. Reviewing math concepts time to time will allow them to master the math concepts.

9) Don’t pressurize:

Slow learner students tend to have lack of confidence, if you pressurize them for time management or anything this will only reduce their confidence.

10) Reward them:

Slow learner students tend to have low confidence. Low confidence impedes anyone’s learning ability. If you reward them time to time this will help them to raise their confidence.

Slow learners are normal students who are simply not interested in studying under traditionally acceptable systems of education. Parents or guardians often do not take their slow learning wards seriously during their earlier days of growth.

Courses like mathematics have great bearing on the child's frame of mind during primary school education. If the concepts are not clear, then the child loses interest in the subject and as the subject has continuity and is dependent on basic principles, this interesting subject becomes stressful for him or her. Subsequently it becomes difficult for parents or guardians or teachers to help such children achieve even passing grades where normal students can achieve almost full marks with little effort.

Self- Check Exercise-2

Q.1 Which of the following is a common strategy used to support slow learners in mathematics?

A) Providing advanced enrichment activities

B) Offering remedial instruction

C) Assigning complex problem-solving tasks

D) Ignoring their individual needs

Q.2 What is a characteristic feature of effective strategies for teaching slow learners in mathematics?

A) Overwhelming them with advanced concepts

B) Focusing solely on rote memorization

C) Providing personalized support and scaffolding

D) Exclusively using one instructional method

Q.3 Differentiated instruction involves _____ teaching methods and materials to meet the specific learning needs of slow learners.

Q.4 Providing _____ feedback and encouragement helps build the confidence of slow learners in mathematics.

Q.5 Slow learners benefit from _____ instruction, breaking down complex concepts into smaller, more manageable steps.

Q.6 Using _____ aids and manipulatives can enhance the understanding of mathematical concepts for slow learners.

8.5 Strategies for Teaching of Mathematics to Learners with Dyscalculia

Dyscalculia is a mathematics-related disability resulting from neurological dysfunction. Students who are diagnosed with Dyscalculia have average to above-average intellectual functioning and a significant discrepancy between their math skills and their chronological-age-peer norms. For a diagnosis of Dyscalculia, it must be determined that the math deficit is not simply related to issues such as poor instruction, vision, hearing or other physical problems, cultural or language differences, or developmental delays.

Teaching mathematics to learners with dyscalculia requires tailored strategies that address their specific learning needs and challenges. Here are some effective strategies for teaching mathematics to learners with dyscalculia:

Multisensory Approach: Incorporate multisensory techniques that engage multiple senses (such as sight, touch, and hearing) to reinforce mathematical concepts. For example, use manipulative, visual aids, and interactive activities to help learners grasp abstract mathematical ideas.

Visual Representation: Provide visual representations of mathematical concepts through charts, diagrams, and models. Visual cues can enhance understanding and facilitate retention for learners with dyscalculia.

Concrete Examples: Use concrete examples and real-life contexts to illustrate mathematical concepts. Relating math to everyday experiences can make abstract concepts more meaningful and accessible to learners with dyscalculia.

Scaffolded Instruction: Break down mathematical tasks into smaller, more manageable steps, and provide clear, step-by-step instructions. Scaffolding helps learners with dyscalculia build confidence and master skills gradually.

Repeated Practice: Offer opportunities for repeated practice and reinforcement of mathematical skills. Regular practice sessions with structured activities can help learners with dyscalculia strengthen their mathematical abilities over time.

Differentiated Instruction: Differentiate instruction to meet the diverse learning needs of learners with dyscalculia. Provide individualized support, accommodations, and modifications based on each learner's strengths, weaknesses, and learning style.

Positive Reinforcement: Provide positive reinforcement and encouragement to boost learners' confidence and motivation. Celebrate their progress and successes, no matter how small, to foster a positive attitude towards mathematics.

Flexible Assessment: Use flexible assessment strategies that accommodate the unique challenges faced by learners with dyscalculia. Offer alternative assessment methods, such as oral assessments or performance tasks, to accurately assess their mathematical understanding.

Collaborative Learning: Encourage peer collaboration and cooperative learning activities. Working with peers can provide additional support and help learners with dyscalculia develop social skills while learning mathematics.

Supportive Environment: Create a supportive and inclusive learning environment where learners with dyscalculia feel valued, respected, and supported. Foster a growth mind set and emphasize the importance of effort and perseverance in learning mathematics.

By implementing these strategies, educators can create an inclusive learning environment where learners with dyscalculia can develop their mathematical skills, build confidence, and achieve success in mathematics.

In Accommodating Math Students with Learning Disabilities, author Rochelle Kenyon lists the following strategies for teaching a student with math-related learning disabilities.

- Avoid memory overload. Assign manageable amounts of work as skills are learned.
- Build retention by providing review within a day or two of the initial learning of difficult skills.
- Provide supervised practice to prevent students from practicing misconceptions and "misrules."
- Make new learning meaningful by relating practice of sub skills to the performance of the whole task.
- Reduce processing demands by pre teaching component skills of algorithms and strategies.
- Help students to visualize math problems by drawing.
- Use visual and auditory examples.
- Use real-life situations that make problems functional and applicable to everyday life.
- Do math problems on graph paper to keep the numbers in line.
- Use uncluttered worksheets to avoid too much visual information.
- Practice with age-appropriate games as motivational materials.
- Have students track their progress.
- Challenge critical thinking about real problems with problem solving.

- Use manipulative and technology such as tape recorders or calculators.

Self- Check Exercise-3

Q.1 What is a common characteristic of students with dyscalculia?

A) Exceptional proficiency in mathematics

B) Difficulty with basic arithmetic operations

C) Advanced problem-solving skills

D) Strong memory for mathematical concepts

Q.2 Which of the following is an effective strategy for teaching students with dyscalculia?

A) Using only traditional instructional methods

B) Ignoring their individual needs

C) Providing visual aids and manipulative

D) Focusing solely on memorization

Q.3 Providing _____ instructions and breaking tasks into smaller steps can help students with dyscalculia better understand mathematical concepts.

Q.4 Students with dyscalculia may benefit from using _____ to represent mathematical concepts and operations visually.

Q.5 Offering _____ practice and repetition can help reinforce mathematical skills for students with dyscalculia.

Q.6 Providing _____ feedback and encouragement is important for supporting the confidence and motivation of students with dyscalculia.

8.6 Problems in Teaching and Learning of Mathematics

Any analysis of mathematics education in our schools will identify a range of issues as problematic. We structure our understanding of these issues around the following four problems which we deem to be the core areas of concern:

1. A sense of fear and failure regarding mathematics among a majority of children,

2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time,

3. Crude methods of assessment that encourage perception of mathematics as mechanical computation, and

4. Lack of teacher preparation and support in the teaching of mathematics.

Fear and Failure

Mathematics anxiety and 'math phobia' are terms that are used in popular literature. In the Indian context, there is a special dimension to such anxiety. With the universalization of elementary education made a national priority, and elementary education a legal right, at this historic juncture, a serious attempt must be made to look into every aspect that alienates children in school and contributes towards their non-participation, eventually leading to their dropping out of the system. If any subject taught in school plays a significant role in alienating children and causing them to stop attending school, perhaps mathematics, which inspires so much dread, must take a big part of the blame. Such fear is closely linked to a sense of failure. By Class III or IV, many children start seeing themselves as unable to cope with the demands made by mathematics. In high school, among children who fail only in one or two subjects in year-end examinations and hence are detained, the maximum numbers fail in mathematics. This statistic pursues us right through to Class X, which is when the Indian state issues a certificate of education to a student. The largest numbers of Board Exam failures also happen in mathematics.

Disappointing Curriculum

Any mathematics curriculum that emphasizes procedure and knowledge of formulas over understanding is bound to enhance anxiety. The prevalent practice of school mathematics goes further: a silent majority gives up early on, remaining content to fail in mathematics, or at best, to see it through, maintaining a minimal level of achievement. For these children, what the curriculum offers is a store of mathematical facts, borrowed temporarily while preparing for tests. On the other hand, it is widely acknowledged that more than in any other content discipline, mathematics is the subject that also sees great motivation and talent even at an early age in a small number of children. These are children who take to quantization and algebra easily and carry on with great facility. What the curriculum offers for such children is also intense disappointment. By not offering conceptual depth, by not challenging them, the curriculum settles for minimal use of their motivation. Learning procedures may be easy for them, but their understanding and capacity for reasoning remain under exercised.

Crude Assessment

We talked of fear and failure. While what happens in class may alienate, it never evokes panic, as does the examination. Most of the problems cited above relate to the tyranny of procedure and memorization of formulas in school mathematics, and the central reason for the ascendancy of procedure is the nature of assessment and evaluation. Tests are designed (only) for assessing a student's knowledge of procedure and memory of formulas and facts, and given the criticality of examination performance in school life, concept learning is replaced by procedural memory. Those children who cannot do such replacement successfully experience panic, and suffer failure. While mathematics is the major ground for formal problem solving in school, it is also the only arena where children see little room for play in answering questions. Every question in mathematics is seen to have one unique answer, and either you know it or you don't. In Language, Social Studies, or even in Science, you may try and demonstrate partial knowledge, but there is no scope for doing so in mathematics. Obviously, such a perception is easily coupled to anxiety.

Inadequate Teacher Preparation

More so than any other content discipline, mathematics education relies very heavily on the preparation that the teacher has, in her own understanding of mathematics, of the nature of mathematics, and in her bag of pedagogic techniques. Textbook-centered pedagogy dulls the teacher's own mathematics activity. At two ends of the spectrum, mathematics teaching poses special problems. At the primary level, most teachers assume that they know all the mathematics needed, and in the absence of any specific pedagogic training, simply try and uncritically reproduce the techniques they experienced in their school days. Often this ends up perpetuating problems across time and space. At the secondary and higher secondary level, some teachers face a different situation. The syllabi have considerably changed since their school days, and in the absence of systematic and continuing education programmes for teachers, their fundamentals in many concept areas are not strong. This encourages reliance on 'notes' available in the market, offering little breadth or depth for the students. While inadequate teacher preparation and support acts negatively on all of school mathematics, at the primary stage, its main consequence is this: mathematics pedagogy rarely resonates with the findings of children's psychology. At the upper primary stage, when the language of abstractions is formalized in algebra, inadequate teacher preparation reflects as inability to link formal mathematics with experiential learning. Later on, it reflects as incapacity to offer connections within mathematics or across subject areas to applications in the sciences, thus depriving students of important motivation and appreciation.

Teaching and learning mathematics can present various challenges for both educators and students. Here are some common problems encountered in the teaching and learning of mathematics, along with suggested measures to address them:

Lack of Engagement: Some students may find mathematics dull or uninteresting, leading to disengagement and low motivation.

Suggestive Measures:

1. Incorporate hands-on activities, real-life examples, and interactive lessons to make mathematics more engaging and relevant.
2. Use technology such as educational apps, online simulations, and interactive whiteboards to enhance student engagement.
3. Provide opportunities for student choice and autonomy in selecting math topics or projects to study.

Difficulty Understanding Abstract Concepts: Mathematics often involves abstract concepts and symbols that can be challenging for students to grasp.

Suggestive Measures:

1. Use concrete manipulative, visual representations, and real-world contexts to illustrate abstract mathematical concepts.
2. Break down complex ideas into smaller, more manageable steps, and provide scaffold support to guide students through the learning process.
3. Encourage students to verbalize their thinking and explain concepts in their own words to deepen understanding.

Fear of Making Mistakes: Some students may have a fear of making mistakes in mathematics, leading to anxiety and reluctance to participate.

Suggestive Measures:

1. Create a supportive classroom environment where mistakes are viewed as opportunities for learning and growth.
2. Encourage a growth mindset by emphasizing the importance of effort, perseverance, and resilience in mathematics.
3. Provide constructive feedback that focuses on the process rather than just the outcome, and celebrate students' efforts and progress.

Limited Problem-Solving Skills: Many students struggle with problem-solving, which is a fundamental aspect of mathematics.

Suggestive Measures:

1. Teach problem-solving strategies explicitly, such as identifying key information, breaking problems into smaller steps, and checking solutions for accuracy.
2. Provide ample opportunities for students to practice problem-solving through a variety of tasks and activities.

3. Foster a collaborative problem-solving culture where students work together to tackle challenging problems and share their strategies and solutions.

Language Barriers: Language barriers can pose challenges for students who are English language learners or who have difficulty understanding mathematical language.

Suggestive Measures

1. Use visuals, diagrams, and hands-on activities to support comprehension for students with limited language proficiency.
2. Provide bilingual support materials or translations of mathematical terms and concepts in students' native languages.
3. Encourage peer collaboration and provide opportunities for students to explain mathematical concepts in their own words.

Unequal Access to Resources: Disparities in access to resources such as textbooks, technology, and extra help outside of class can exacerbate inequities in mathematics education.

Suggestive Measures

1. Advocate for equitable funding and resources to ensure that all students have access to high-quality mathematics instruction and support.
2. Provide additional support and resources, such as tutoring, mentoring, or after-school programs, for students who need extra help.
3. Foster partnerships with community organizations, businesses, and local universities to expand access to resources and opportunities for students.

By addressing these challenges and implementing suggested measures, educators can create more inclusive, engaging, and effective mathematics learning environments where all students have the opportunity to succeed.

More Suggestive Measures

While the litany of problems and challenges magnifies the distance we need to travel to arrive at the vision articulated above, it also offers hope by way of pointing us where we need to go and what steps we may/must take. We summarize what we believe to be the central directions for action towards our stated vision. We group them again into four central themes:

1. Shifting the focus of mathematics education from achieving ‘narrow’ goals to ‘higher’ goals,

2. Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging mathematician,

3. Changing modes of assessment to examine students' mathematisation abilities rather than procedural knowledge,

4. Enriching teachers with a variety of mathematical resources.

Self- Check Exercise-4

Q.1 What is a common problem faced by students in learning mathematics?

A) Lack of resources

B) Fear of making mistakes

C) Excessive homework

D) Limited class time

Q.2 What is a suggested measure to address language barriers in mathematics education?

A) Providing bilingual support materials

B) Increasing class size

C) Eliminating word problems

D) Ignoring the issue

Q.3 Limited _____ skills can hinder students' ability to solve mathematical problems effectively.

Q.4 Providing a supportive classroom environment where mistakes are viewed as opportunities for learning can help alleviate students' _____ of making errors in mathematics.

Q.5 Unequal access to resources such as textbooks and technology can exacerbate _____ in mathematics education.

Q.6 Encouraging a growth mind set and emphasizing the importance of effort and perseverance can help foster a positive attitude towards _____ in students.

8.7 Summary

In teaching and learning mathematics, various challenges can arise, including student disengagement, difficulty understanding abstract concepts, fear of making

mistakes, limited problem-solving skills, language barriers, and unequal access to resources. To address these challenges, educators can implement strategies such as incorporating hands-on activities, providing visual representations, fostering a growth mindset, teaching problem-solving strategies, offering bilingual support, and advocating for equitable resources. By creating inclusive, engaging, and supportive learning environments, educators can help all students develop confidence, proficiency, and a positive attitude towards mathematics.

8.8 Glossary

Gifted Learners: Students who demonstrate exceptional abilities, talents, or potential in one or more areas, including mathematics.

Slow Learners: Students who require additional time, support, or instruction to master academic concepts and skills compared to their peers.

Dyscalculia: A specific learning disorder characterized by difficulties with mathematical skills, including understanding numbers, concepts of quantity, mathematical operations, and mathematical reasoning.

Enrichment: Providing advanced or supplemental learning opportunities beyond the standard curriculum to challenge and engage gifted learners in mathematics.

8.9 Answers to Self- Check Exercise

Self- Check Exercise - 1

Answer1: B) Offering enrichment opportunities

Answer2: D) Tailoring instruction to their advanced abilities

Answer3: independent research

Answer4: customizing or tailoring

Answer5: collaborate or interact

Self- Check Exercise - 2

Answer6: challenging or complex

Answer1: B) Offering remedial instruction

Answer2: C) Providing personalized support and scaffolding

Answer3: adapting or customizing

Answer4: positive or constructive

Answer5: scaffold or supported

Answer6: visual or concrete

Self- Check Exercise-3

Answer1: B) Difficulty with basic arithmetic operations

Answer2: C) Providing visual aids and manipulative

Answer3: explicit or clear

Answer4: diagrams or models

Answer5: structured or guided

Answer6: positive or constructive

Self- Check Exercise-4

Answer1: B) Fear of making mistakes

Answer2: A) Providing bilingual support materials

Answer3: problem-solving

Answer4: fear

Answer5: disparities

Answer6: mathematics

8.10 References / Suggested Readings

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8.11 Terminal Questions

Q.1 Explain the Strategies for Teaching of Mathematics to Gifted Children